Iran University of Science and Technology

International Journal of

Railway Research

ISSN: 2423-3838

Developing Passenger Hub Location Problem Based on Origin-Destination Trips Derived by Gravity Model (Case Study on Iran's Rail Network)

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ARTICLE INFO

Article history:
Received: 3.08.2018
Accepted: 8.10.2018
Published: 15.12.2018

Keywords:
Rail Transportation
Hub Location Problem
Trip Distribution
Mathematical Programming

A B S T R A C T

Iran’s geographical situation, in particular existing long-distance routes, makes rail transportation more attractive and efficient rather than the other modes of public transportation. Transport costs as well as passenger utility are main concerns for local and national authorities. Therefore, Hub location is an important problem in transport planning in which transport industries are looking for selected points for passengers to change trains from their origins to destinations. In this paper, a mathematical model for solving hub location problem is developed and validated on Iranian rail network. Passenger supplies and demand from origin to destination nodes are estimated using the well-known model of Gravity as well as the shortest path algorithm that is utilized for route assignment. Running the proposed procedure and model by GAMS revealed that choosing appropriate hubs, even in less sophisticated connectivity rail network, can significantly reduce the transport cost and enhance rail transport productivity over the rail network.

1. Introduction

Hubs are known as essential facilities that serve for switching and transshipment in distribution systems. Instead of direct transferring passengers on each origin-destination pairs, the above facilities concentrate and dispatch passenger or cargo flows in order to minimize total transport cost. In the overall concept of hub location problems, passenger or cargo flows from the same origin to different destinations are carried to the hub and combined with flows that have different origins which are sent to the same destination [1]. Due to extreme transport costs, hub location problems arise from transport networks when direct transport of goods and passengers between origin-destination pairs it is not desirable. The location of hubs has recently become an important research area in the field of location modeling [2]. A hub network is alternatively used where hubs act as collection, consolidation, transferring and distribution points. The main advantage of exploiting a hub network is to decrease transport costs between hubs, which leads to reducing overall transport costs on the network. Origin and destination nodes can be connected to one or more hubs depending on constraints, rail, and road or connection network. In general, hub location problems involve two decision making tasks: 1) Choosing hubs to establish from the given set of potential hubs, and 2) Allocation of non-hub nodes to established hubs [3].

1.1. Transport Planning

There are four main stages of transport planning including trip generation, trip distribution, modal split and eventually route assignment. Trip generation measures the

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frequency of trips to travel. Trips are also represented as trip starts known as production and consequently ends known as attraction. Trip productions are distributed to match the trip attraction underlying travel time and/or cost, resulted to tables of person-trip demands, known as trip distribution. Trip tables are then essentially factored to reflect relative proportions of trips by alternative modes, named as modal split. Finally, trip tables are assigned to selected available routes over the network, called as route assignment [4]. One of the most important issues on hub location problem is to distribute trips over a set of origins and destinations.

1.2. Gravity Model

The Gravity model is a well-known technique for trip distribution, where the objective is to recombine trip ends from trip generation into trips, typically defined as production-attraction pairs and not origin-destination pairs. The trip distribution model is essentially a destination choice model and generates a trip matrix (or trip table) T_{ij} for each trip purpose utilized in the trip generation model as a function of activity system attributes (indirectly through the generated productions Pi and trip attractions A_j) regarding to network attributes typically impersonal travel times [4]. The general form of the trip distribution model as the second step of the transport planning is defined as Equation (1) where t_{ij} represents a measure of travel impedance (travel time or generalized cost) between the two zones i and j as well as p describes trip purpose. For internal trips, perhaps the most common model is the so-called gravity model shown as Equation (2) where a_i and b_j are represented by Equations (3) and (4) and $\mathrm{f}(t_{ij})$ is the function of network level of service (4) known also as impedance function.

$$T_{ij} = f_{TD}(A, t_{ij}) = f_{TD}(P_i, A_j, t_{ij}) \quad (1)$$

$$T_{ij} = a_i b_j P_i, A_j \mathrm{f}(t_{er}) \quad (2)$$

$$a_i = \left[\sum_j b_j A_j \mathrm{f}(t_{ij})\right]^{-1} \quad (3)$$

$$b_j = \left[\sum_i a_i p_i \mathrm{f}(t_{ij})\right]^{-1} \quad (4)$$

The old developed model minimized all distances to the central facility from desired origins [5]. In 1985, it was concluded that if domestic airlines centralized, the hub profit will increase and also monopoly can increase the profit [6]. Later formulation for hub location problem based on quadratic integer programming was presented by [7] and developed by Campbell in 1992 when he found the best location for the facilities to decrease the transportation costs and considered different situations [8]. He also examined 4 discrete models of hub location problems; including p-hub median problem, incapacitated hub location problem, p-hub center problem and hub covering problem by developing integer programming to solve the descript models while more improvement addressed in 1994 [9]. In 1996, a multiple allocation has been conducted to more than one hub to decrease the total transport cost. Integer programming models were developed and solved by two heuristic solutions and concluded that the heuristic algorithms had superior performance [10]. At the same time, a new LP formulation was presented with fewer variables and constraints and solved the model with a heuristic algorithm. By using the required data it was concluded that the formulation can represent the optimum results in less time and showed that this model needs lower RAM on PC and as the result, bigger problems can be solved [11]. In 2003, a paper on location models for airline hubs was published and explained that due to the computational complexity of the formulation; the model was solved using a heuristic algorithm based on Tabu search and a data set to minimize the total fixed and transportation costs [12]. In 2008 another paper about network hub location problem was presented in which authors reviewed the hub location problems and made categories for them [1]. Following that in 2009, Sim et al. solved the stochastic p-hub center problem with service-level constrains. They used heuristic algorithms and found that the optimal locations of the p-hubs tend to form a certain structure, where one hub is located in the center of the service region with the remaining hubs located around this central hub [13]. Stochastic air freight hub location and flight routes planning were later studied in 2009 in which the model was separated into two decision stages. The first stage, which was the decision not affected by randomness, determines the number and the location of hubs. The second stage, which was the decision affected by randomness, determines the flight routes to transport flows from origins to destinations based upon the hub location and realized uncertain scenario [14]. They tested the model with experimental data from Taiwan and China and concluded that the model works well when the demand is changing. They also concluded that the stochastic models works better than the similar certain models [14]. A quadratic formulation for hub location problems was also presented in 2011 and made to linear model in which results showed that linearization of the model works efficiently in terms of processing time [15]. At the same time, Contreras et.al worked on a stochastic incapacitated hub location problem. They assumed that demand and transportation costs are stochastic and used a Monte-Carlo simulation [16]. In 2012, another study conducted on capacitated p-hub median problem with integral constraints. A genetic algorithm was used for solving the model and the results showed the crowded airports are better hub candidates rather than low traffic ones [17]. Alumur and coworkers have also studied on Hub location under uncertainty. Two sources of uncertainty were considered: the set-up costs for the hubs and the demands to be transported between the nodes. Generic models were presented for single and multiple allocation versions of the problems. Firstly, the two sources of uncertainty were analyzed separately and afterwards a more comprehensive model was developed considering all sources of uncertainty. Using a set of computational tests, they analyzed the changes in the solutions driven by the different sources of uncertainty considered as isolated and combined [18]. Hub location problems and review of models, classification, solution techniques, and applications was studied in 2013 in which some of the hub location models from 2007 to 2013 were reviewed in detail [19]. In 2014 a study on the incapacitated single allocation hub location problem represented using genetic algorithm. The main objective was to minimize the total transportation costs. It was concluded that solving hub location problem with genetic algorithm is acceptable but the time of computation is a little bit longer [20]. At the same time, a two-stage stochastic programming approach for hub location problem under uncertainty was examined. They assumed demand and transportation costs as uncertain factors and they used Iran's air network data to test their formulation [21].

1.3. Research Vision

The main aim of this research work lies on locating Hubs in rail transportation when trip distribution is simultaneously estimated by Gravity model. In this case, trip generations are used as input data for modeling and formulation instead of origin-destination matrix for passengers. Impedance factor is distance which is a very important factor in railway transport in which passengers usually set their trips according to distances. So, a two-stage procedure is developed to estimate the trip distribution table at the first and selecting hubs for changing lanes over the network at the second stage.

2. Procedure and Modeling

2.1. Research Procedure

As aforesaid, the main aim of this study is to find the best location for Hubs in intercity railway network. It is assumed that all nodes with specified population can be selected as origin, destination or both. It means that all populated areas or cities that are connected to railway network may have the potentiality of supply and demand on production and trip attractions. Some of the nodes are nominated as hub candidate. It is also assumed that the hubs may have supply and demand, so the shortest paths between all origins, destination and hubs have been determined utilizing the Floyd's algorithm [22]. Populations for all areas are now assigned according to published stats available in Iran's statistics center reports [23]. Supply and demand for all nodes have been assigned utilizing the gravity model assuming population as gravitation criterion. After developing and running the model on experimental data, the best locations for the hubs are determined where the number of trains and wagons are assigned to each O-D pairs. In other words, the proposed model determines the number of wagons that should be assigned to carry passengers directly from each origin to its corresponding destinations and determining the best locations for Hubs, as well.

2.2. Mathematical Modeling

The mathematical model developed in this research work consists of the objective function, decision variables, parameters, and relevant constraints. Here are indices and parameters defined for modeling the problem.

Identifier i defines origins (i=1,2,…,n),

j defines destinations (j=1,2,…,m)

k defines selected hubs as candidate (k = 1, 2, …, K).

P is a scalar that indicates the number of wagons that may be coupled to the train. There are three kinds of trains consisting of 8, 10 and 12 wagons.

B_P is the capacity, the number of passengers in each train including 320 for 8-wagon train and 400 and 480 passengers for 10 and 12-wagon trains, respectively.

HP_{k} is a binary variable which defines Hubs. If node K is selected as a Hub, then HP_{k}=1 otherwise HP_{k}= 0.

The parameter PH is a scalar that shows the maximum number of hubs that can be selected over the rail network.

M is a large number that is used for the mathematical modeling [22].

T_1 is a scalar that shows the passenger kilometer rate.

T_2 is the fixed cost of running a wagon.

T_3 is a scalar that represents the variable cost of running a wagon.

T_4 is the fixed cost of empty chairs or the number of travelers that are left from the trip.

T_5 is the variable costs of empty seats.

O_{i} is an array that represents the passenger flow from node "i". Flow means the number of passengers in node "i" who are boarding the train to travel.

W_j is another array that represents the passenger flow to node j.

OD_1 defines the shortest distances between all the nodes that are tabulated as a square matrix in which the numbers on its main diagonal is 0.

OD_2 and OD_3 have the same definitions for the shortest paths between origins to hubs and hubs to destinations, respectively.

UP_i is a variable which represents the number of passengers that were not able to be on board in origin i.

UC_i is the number of empty seats in the origin i.

UK_k is the variable which defines the number of passengers that were not able to be on board in Hub k.

CK_k is the number of empty seats in Hub k.

Z_{ikp} is an integer variable that defines the number of trains composing of P-wagons from origin i to Hub k.

$X_{\mathrm{kj}p}$ and Y_{ijp} are the same integer variables which define the number of P-wagon trains from hub k to destination j and the number of P-wagon trains from origin i to destination j, respectively.

2.3. Objective Function and Constraints

Since the global objective function is to maximize the total profits, it can be calculated by subtraction of all costs from the total revenues or incomes. The revenue is calculated by multiplying the passenger kilometer fee (rate) to the total passenger kilometer traveled by trains. Costs consist of two main parts. The first part is the setup cost and the second is the cost of empty seats and travelers that have been left behind. So, the objective function is formulated as Equation (5), where G1, G2 and G3 will be defined more in detail.

$$Max\ Z = T_1 \times G_1 - G_2 - T_4 \times G_3 \qquad (5)$$

The second equation corresponds to the total revenue. Revenue is obtained by multiplying P-Wagon train kilometer in train capacity, both multiplied to passenger kilometer rate, which is represented by national currency. P-Wagon train kilometer is now determined by multiplying the number of P-Wagon trains in each path to the specific distance. Therefore, the first element of objective function, G1, is formulated by Equation (6).

The third equation is performed to calculate the P-Wagon train cost. P-Wagon train cost consists of two main parts including the fixed and the operation costs.

$$G_1 = \sum_i \sum_j \sum_p OD_{1_{ij}} \times Y_{ijp} \times B_p + \sum_i \sum_k \sum_p OD_{2_{ik}} \times Z_{ikp} \times B_p + \sum_k \sum_j \sum_p OD_{3_{kj}} \times X_{kjp} \times B_p \quad (6)$$

The fixed cost is constant but the operation cost is dependent on the train traveled distance and the number of wagons (P). So, it is calculated by multiplying the variable cost rate to train traveled kilometer formulated by Equation (7):

$$G_2 = \sum_i \sum_j \sum_p (T_2 + T_3 OD_{1_{ij}} \times P - Val) \times Y_{ijp} + \sum_i \sum_k \sum_p (T_2 + T_3 OD_{2_{ik}} \times P - Val) \times Z_{ikp} + \sum_k \sum_j \sum_p (T_2 + T_3 OD_{3_{kj}} \times P - Val) \times X_{kjp} \quad (7)$$

The forth equation considers of the cost of empty seats and the unsatisfied passengers. Since, hubs can be used as origins and destinations, the costs of empty seats and the unsatisfied passengers is formulated by using Equation (8):

$$G_3 = \sum_i UP_i + T_5\, UC_i + \sum_k UK_k + T_5\, CK_k \quad (8)$$

There is a constraint for the number of hubs so, the Equation (9) is required to guarantee that the number of hubs in the network doesn't exceed of the predefined scalar PH.

$$\sum_k HP_k \leq PH \quad (9)$$

The sixth constraint is added to guarantee that the total number of the passengers in each origin should be equal to the total number of passengers who travel to their destinations. It means that passengers that move from origin to the hub or directly move to the destination should be balanced with empty seats and the unsatisfied passengers and total traveling demand in each origin. Equation (10) represents the above consideration.

$$\sum_k \sum_p B_p \times Z_{ikp} + \sum_j \sum_p B_p \times Y_{ijp} + UP_i - UC_i = O_i \quad (10)$$

Adding the seventh constraint is to guarantee all attractive demands to destinations are satisfied, that is formulated by Equation (11):

$$\sum_k \sum_p B_p \times X_{kjp} + \sum_i \sum_p B_p \times Y_{ijp} \geq W_j \quad (11)$$

The eighth equation is for balancing the constraint. It means that all entering passengers to each hub should be equal to the polling out passengers from that which is formulated by Equation (12). Equations (13) and (14) are the large M constraints which control the assigned value of HPk while node K is used as a hub.

$$\sum_i \sum_p B_p \times Z_{ikp} - \sum_j \sum_p B_p \times X_{kjp} - UK_k + UC_k = 0 \quad (12)$$

$$M \times HP_k \geq \sum_i \sum_p Z_{ikp} \quad (13)$$

$$M \times (HP_k - 1) < \sum_i \sum_p Z_{ikp} \quad (14)$$

Finally, constrain (15) sets the states of a binary and integer variables used for modeling.

$$HP_k \in \{0,1\}\ \forall k \& Z_{ik}, X_{kj}, Y_{ij} \in integer \quad (15)$$

3. Experimental Analysis and Discussion

For applying the mathematical model, the Iranian rail network is selected as shown in Figure 1. The main train stations are Azarbaijan, North West, Tehran, North East, Khorasan, Qom, Isfahan, Yazd, Kerman, South East, Hormozgan, Fars, East, Arak, Lorestan, Zagros, South and North, mainly stated by their city names.

Figure 1. Railway map of case study area (Source of Image: http://www.iranrail.net)

The proposed model is run by using the data published by, Iran's rail network company, the rail branch of the Iranian Ministry of Roads and Urban Planning [24].

Because data of travel demand was not available, the demand data is calculated by using the gravity model. The gravity for trip demand is formulated as Equation (16), where P_i represents the population of origin i, P_j represents the population of destination j and S_{ij} is the travel supply from origin i to destination j. α is a coefficient and D_{ij} is the distance from origin i to destination j. In this study α is computed and is equal to $9.7e-10$ by using the well-known technique of the least square errors.

$$Demand_{ij} = \alpha \frac{P_i P_j S_{ij}}{D_{ij}^2} \quad (16)$$

The population of all origins and destinations were extracted from the data that are released by the Iran's statistical center [25] and tabulated in Table 1. Utilizing the gravity model, by using Equation (16), the travelling demands are obtained and are presented in Table 2.

Table 1. Population in origins and destinations [25]

Area	population	Area	population
Azarbaijan	7174871	Isfahan	5120850
North West	2331222	Yazd	1138533
Tehran	13267637	Kerman	3164718
North East	702360	South East	2775014
Khorasan	6434501	Hormozgan	1776415
Qom	1292283	Fars	4851274
East	768898	Zagros	580158
Arak	1429475	South	4710509
Lorestan	1760649	North	5152401

Five stations of Tehran, North East, Qom, Isfahan and Yazd have been nominated as hubs. The model was run following the above assumptions by using the demand and supply extracted from the experimental data. Results showed that, if the maximum number of hubs is three, North East, Yazd and Isfahan stations are optimally selected. In this case, origins to hubs and hubs to destinations are also obtained by the using the model in a scale of weekly P-wagon trips. The results in details are presented in Appendix 1.

Table 2. Annual passenger demand and supply

Area	Supply	Demand
Azarbaijan	960561	1703277
North West	1499507	2649135
Tehran	8064458	2214781
North East	859624	501731
Khorasan	7065694	860446
Qom	420528	4980116
Isfahan	645582	2318227
Yazd	533328	366680
Kerman	389802	381514
South East	67601	212490
Hormozgan	587592	271643
Fars	146093	1941704
East	233708	202824
Arak	294432	1025712
Lorestan	738482	796821
Zagros	723350	620101
South	1661675	1061165
North	621103	3404748

For the data that are presented in Appendix 1 the trips are summarized into three types of trains including 8, 10 and 12 wagons. For example, the results of the model require that the passengers from the North West to Yazd need a plan to be transferred by using 63 trains composed of 12 wagons, in every week. The results also extract the outgoing trips from hubs where hubs send passengers to destinations. For example, the weekly number of 12-Wagon trains traveling from the North West to Yazd Hub is 63 while the weekly number of 10-Wagon trains traveling from South to North East Hub that is one.

The number of passengers who are not able to be on board, that are known as unsatisfied passengers, and empty seats provided from each origin are also derived by the model and is presented Table 3. As shown, model has assigned unsatisfied passengers or empty seats for origins but both are practically impossible.

Table 3. Unsatisfied passengers and empty seats in origin per week (Hubs = 2)

Row	Origin	Unsatisfied Passengers	Empty Seats
1	Arak	128	0
2	Azarbaijan	12	0
3	East	0	126
4	Fars	0	438
5	Hormozgan	0	248
6	Isfahan	0	48
7	Kerman	116	0
8	Khorasan	0	46
9	Loresatan	0	110
10	North	0	58
11	North East	0	88
12	North West	0	250
13	Qom	0	230
14	South	114	0
15	South East	0	88
16	Tehran	10	0
17	Yazd	106	0
18	Zagros	68	0

4. Sensitivity Analysis

If the number of hubs is restricted to 2, the North East and Yazd will be selected by the model. The other cases, implemented as sensitivity analysis, are presented in Table 4 in which the hubs are assigned as origin denoted by O or destination denoted by D. If the number of hubs is increased to 5, Tehran, North East, Qom, Isfahan and Yazd will be selected by the model.

As shown in Table 4, by selecting two hubs, Yazd works as an origin with the weekly number of 12-Wagon trains traveling from Yazd to all the destinations that is 597, as well as the weekly number of 10-Wagon trains that is 69. The weekly number of 12-Wagon trains traveling from the North East Hub to the other destinations is 367. The number of passengers that were not able to be on board in the origins and the number of empty seats can also be computed by running the model. This is briefly presented in Table 5.

Table 4. Number of trains allocated to Hubs

Hubs	No. Origins	Hub as Origin or Destination	No. of Destinations	Weekly P-Wagon trains 8	10	12
2	-	Yazd (O)	11	6	69	597
	7	Yazd (D)	-	204	200	335
	-	North East (O)	12	8	3	367
	14	North East (D)	-	25	27	344
3	-	Yazd (O)	5	0	0	464
	8	Yazd (D)	-	100	100	314
	-	North East (O)	13	7	1	317
	11	North East (D)	-	0	1	345
	-	Isfahan (O)	12	2	3	247
	2	Isfahan (D)	-	100	100	101
5	-	Yazd (O)	6	0	0	451
	6	Yazd (D)	-	101	104	297
	-	North East (O)	11	10	1	350
	13	North East (D)	-	14	7	342
	-	Isfahan (O)	8	3	1	247
	1	Isfahan (D)	-	100	100	100
	-	Tehran (O)	1	0	0	7
	1	Tehran (D)	-	0	0	7
	-	Qom (O)	1	1	1	0
	1	Qom (D)	-	0	0	1

Table 5. Global stats on different Hubs

Number of Hubs	0	2	3	5
Profit (E10 *)	2.80	5.22	5.31	5.42
Number of Trains	1092	2231	2208	2245
Empty Seats	1970	1792	1730	1584
Unsatisfied Passengers	154	82	554	88

* Unit: Iranian Currency

As shown in Table 5, the total profit is increased and the number of trains as well, when the number of hubs is increased. On the other hand, empty seats are smoothly constant while unsatisfied passengers are different in different scenarios.

5. Conclusions

Undoubtedly, the Hub locations is one of the most important issues for the transport planning. Therefore, the main aim of this study is to find the best locations as Hubs in the rail network. The procedure includes the estimation of the supply and demand of the transport by using the gravity model that is utilized for the trip distribution. The proposed procedure that is developed in this study comprises of some steps. The first step is to estimate the supply and demand for each candidate nodes over the rail network by utilizing the gravity model followed by developing a mathematical model for locating the best Hubs over the rail network at the second step. The above mentioned steps are then followed by selecting the Iran's rail network as a case study. The Floyd algorithm has been utilized for finding the shortest paths over the network. The well-known software of GAMS has been used for solving the proposed model. The results showed that using five Hubs can increase the total profit. Sensitivity analysis revealed that by choosing five hubs, the number of empty seats and unsatisfied passengers decreases and the overall profit increases.

Research for further studies in this topic is recommended to focus on the existing competitive modes of transportation. Also, time scheduling over the railway transportation network by considering work trips is suggested.

Declaration of Conflicting Interests

The authors declared that no funding, scholarship, or fellowship has been gained to conduct this research work. Therefore, there is no potential conflict of interest with respect to the research, authorship, and publication of this article.

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Appendix 1:

Number of P-Wagon trains allocated in each path

Row	Origin	Hub	Destination	P-Wagon trains in week		
				8	10	12
1	North West	Yazd	-	0	0	63
2	Azarbaijan	Yazd	-	0	0	40
3	Tehran	North East	-	0	0	86
4	Tehran	Yazd	-	100	100	100
5	North East	Yazd	-	0	0	36
6	Khorasan	Isfahan	-	100	100	100
7	Khorasan	Yazd	-	0	0	40
8	Qom	North East	-	0	0	18
9	Isfahan	North East	-	0	0	27
10	Yazd	North East	-	0	0	22
11	Kerman	North East	-	0	0	16
12	South East	Yazd	-	0	0	2
13	Hormozgan	North East	-	0	0	25
14	Fars	Yazd	-	0	0	7
15	East	North East	-	0	0	10
16	Arak	North East	-	0	0	12
17	Lorestan	North East	-	0	0	30
18	Lorestan	Isfahan	-	0	0	1
19	Zagros	North East	-	0	0	30
20	South	North East	-	0	1	69
21	North	Yazd	-	0	0	26
22	-	North East	Khorasan	0	0	2
23	-	North East	Qom	0	0	7
24	-	North East	Isfahan	0	1	95
25	-	North East	Yazd	2	0	0
26	-	North East	Kerman	0	0	16
27	-	North East	South East	0	0	9
28	-	North East	Hormozgan	2	0	0
29	-	North East	Fars	0	0	81
30	-	North East	East	0	0	1
31	-	North East	Arak	0	0	42
32	-	North East	Lorestan	1	0	21
33	-	North East	South	2	0	41
34	-	Isfahan	North West	0	2	9
35	-	Isfahan	North East	0	0	19
36	-	Isfahan	Khorasan	0	0	34
37	-	Isfahan	Qom	1	0	100
38	-	Isfahan	Yazd	0	0	14
39	-	Isfahan	Hormozgan	0	0	10
40	-	Isfahan	East	0	0	6
41	-	Isfahan	Arak	0	1	0
42	-	Isfahan	Lorestan	1	0	11
43	-	Isfahan	South	0	0	1
44	-	Isfahan	North	0	0	42
45	-	Yazd	North West	0	0	100
46	-	Yazd	Azarbaijan	0	0	71
47	-	Yazd	Tehran	0	0	93
48	-	Yazd	Qom	0	0	100
49	-	Yazd	North	0	0	100
50	Khorasan	East	-	1	1	0
51	Khorasan	Zagros	-	0	0	3
52	South East	South	-	0	0	1