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# Two Mathematical Models for Railway Crew Scheduling Problem

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**A B S T R A C T**

Railway crew scheduling problem is a substantial part of the railway transportation planning, which aims to find the optimal combination of the trip sequences (pairings), and assign them to the crew complements. In this problem, each trip must be covered by at least one pairing. The multiple-covered trips lead to impose useless transfers called "transitions". In this study, we have proposed a new mathematical model to simultaneously minimize both costs of trips and transitions. Moreover, a new mathematical model is proposed to find the optimal solution of railway crew assignment problem. This model minimizes the total cost, including cost of assigning crew complements, fixed cost of employing crew complements and penalty cost for short workloads. To evaluate the proposed models, several random examples, based on the railway network of Iran were investigated. The results demonstrated the capability of the proposed models to decrease total costs of the crew scheduling problem.

**Keywords:** Railway, Crew Scheduling Problem, Assignment, Transition Reduction, Workload.

## 1. Introduction

Railway crew scheduling is considered as a substantial part of the railway transportation planning. The aim of this problem is to find the optimal combination of the trip sequences, by which, the whole trips of the fleet timetable are covered with lowest price (Caprara et al., 1999). Different restrictions and rules of the trains and the railway network may cause remarkable complexity for the crew scheduling problem (Kroon et al., 2009). Because crew wages are considered as one of the main costs associated to the railway transportation system, efficient crew schedules can lead to accumulated savings for the system (Hanafi & Kozan, 2014). Crew scheduling cost includes: cost of crew hiring, cost of dispatching crew complements to the missions, cost of crew inhabitancy outside their homes, cost of crew transport from home depots to the other depots, and etc. A small improvement in crew scheduling can lead to huge savings in annual costs of the railway system, which can justify the competitiveness and profitability of the railway system for the operational companies. Therefore, optimizing the railway crew scheduling problem is of interest, in order to reduce the operational costs and increase the profitability of the system.

The railway crew scheduling problem is based on the train timetable. In this timetable, information and some specifications of the trips are presented (Shen et al., 2017; Tamannaei et al., 2016). Each trip has five specifications, which are considered as input parameters of the crew scheduling problem: home depot, destination depot, trip starting time, trip ending time and hour value (cost) of the trip. A sequence of two or more trips is named as a pairing.

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A pairing that can satisfy all constraints of the problem (like start-time, end-time, start-depot and end-depot constraints), is called a feasible pairing. In a railway crew schedule, all trips of the train timetable must be covered by the feasible pairings. At the first phase of the railway crew scheduling problem, it is necessary to find a set of pairings, which covers all trips at minimal cost. These pairings compose the optimal solution and are considered as the optimal pairings. At the second phase of the railway crew scheduling problem, the optimal pairings must be assigned to the crew complements. In this study, the second phase is named as crew assignment.

## A. Previous Studies

During recent years, many studies have been conducted for modelling and solving crew scheduling problem in railway system. Some of these studies are as follows: Caprara et al. (1998) investigated the crew scheduling problem. The objective function of the mathematical model was to find the minimum number of crew in each depot to cover the whole pairings. In another study from Caprara et al. (1999) railway crew scheduling problem were studied, with the aim of reducing operating costs and the number of crew required on Italian railways. In their study, all feasible pairings were produced at first. Then, they selected the optimized pairings to reduce costs and minimize the required number of crew complements. The mathematical model proposed in their paper was based on a set covering problem. Ernst et al. (2001) studied crew planning in Australia railways. They noted that Australia railway network covers long-distance trips, with longer travel times rather than some railways like European railway systems. They proposed a new method appropriate for Australia conditions. Caprara et al. (2001) proposed an effective technique for integrating the pairing generation and crew assignment phases into a unique phase. Freling et al. (2001) solved crew scheduling problem by dividing the price for Dutch Railways. In another study, Freling et al. (2003) proposed a model to simultaneously solve both crew scheduling problem and train scheduling problem. They showed that by simultaneous consideration of these two problem can lead to significant benefits for the operational system. POURSEYED and SALAHI (2006) solved railway crew (conductors) programming using roundtrip and roster algorithms. The purpose of their algorithm was to minimize the number of working days, which corresponds to minimize the number of crew complements needed. Abbink (2008) evaluated crew scheduling problem at Dutch railways. They proposed methods to divide the problem into smaller parts. The results showed that the proposed method could decrease the annual cost of the system, by saving approximately six million Euros per year. M Yaghini and GHANADPOUR (2010) proposed heuristic model for planning railway crews. In their model, the trips that begin and end at the crew home depot were determined. Then, using genetic algorithm, a subset of the pairings that covers all trips with minimal cost, was allocated to the crew complements. Nishi et al. (2011) solved railway crew scheduling problem with column generation method. In this study, the dual inequality was used to reduce computational times. Juette and Thonemann (2012) used divide-and-price method for modelling crew scheduling problem. They divided the whole region into smaller units. In each subregion, trips were covered individually with possible pairings, but overlapping between subregions was allowed. Shijun et al. (2013) studied crew scheduling problem in China railway. The complexity of their study was due to specific rules of Chinese railways. Based on these rules, it is required to consider time periods for rest of the crew implements. The aim of their study was to reduce the number of shifts according to Chinese railway rules. Chen and Shen (2013) presented a mathematical model for the crew scheduling through set covering problem. then, they solved the problem, using column generation method. In a study from Shen et al. (2013), crew scheduling problem in public transportation have been modeled and solved by using genetic algorithm. Hanafi and Kozan (2014) proposed a mathematical model to solve the crew scheduling problem with binary variables. They showed that the exact procedure is very time-consuming and complex. So, to find the good solutions in a reasonable time, heuristic methods like Simulated Annealing (SA) was used. Masoud Yaghini et al. (2015) developed a mathematical model based on the set covering problem, to formulate the the multi-depot train driver scheduling problem in Iranian railways. Their study was in two phases: pairing generation to build all feasible pairings, and pairing optimization to assign the best possible pairing to each train. To solve the problem, a matheuristic by combining a tabu search metaheuristic and a new neighbourhood structure was proposed. Peng and Shen (2016) presented a new shift evaluation approach for railway crew scheduling problem. To solve the problem, an evolutionary algorithm based on genetic algorithm (GA) was proposed, Experiments show the capability of the proposed approach to generate the schedules better than the best-known solutions. Zhou et al. (2016) focused on urban rail crew scheduling problem and presented a new mathematical model to minimize both the related costs of crew workloads and the variance of workload time spreads. They solved the model by an ant colony algorithm which is based on ant travel path choosing strategy. The performances were assessed by conducting case study on Changsha urban railways. Hoffmann et al. (2017) focused on multi-period railway crew scheduling problem with attendance rates for conductors. They proposed a new model based on set covering with several essential restrictions. To solve the problem, a hybrid column generation approach was applied, on the basis of genetic algorithm. They showed the efficiency of the proposed approach by a case of German rail passenger network.

The review of the literature on crew scheduling problem shows that according to the requirements and conditions in different countries, a variety of the models have been proposed for crew scheduling problem. In Table 1, a brief review of some works performed on crew scheduling problem is presented.

Table 1. a brief review of the works studied on crew scheduling problem

| Authors | Finding Optimal Pairings | | Crew Assignment | Solution Method | |
|---|---|---|---|---|---|
| | Set Covering model | Other models | | Exact | Non-exact |
| Caprara et al. (1998) | ✗ | ✓ | ✓ | ✓ | ✗ |
| Caprara et al. (1999) | ✓ | ✗ | ✓ | ✓ | ✗ |
| Freling et al. (2001) | ✓ | ✗ | ✗ | ✓ | ✗ |
| Ernst et al. (2001) | ✗ | ✓ | ✓ | ✓ | ✗ |
| POURSEYED and SALAHI (2006) | ✗ | ✓ | ✓ | ✗ | ✗ |
| M Yaghini and GHANADPOUR (2010) | ✓ | ✗ | ✗ | ✗ | ✗ |
| Guillermo and José (2009) | ✓ | ✗ | ✗ | ✗ | ✓ |
| Kwan (2011) | ✓ | ✗ | ✗ | ✓ | ✗ |
| Jütte et al. (2011) | ✓ | ✗ | ✗ | ✓ | ✗ |
| Juette and Thonemann (2012) | ✓ | ✗ | ✗ | ✓ | ✗ |
| Chen and Shen (2013) | ✓ | ✗ | ✗ | ✓ | ✗ |
| Shijun et al. (2013) | ✓ | ✗ | ✗ | ✗ | ✓ |
| Hanafi and Kozan (2014) | ✗ | ✓ | ✗ | ✗ | ✓ |
| Masoud Yaghini et al. (2015) | ✓ | ✗ | ✓ | ✗ | ✓ |
| Peng and Shen (2016) | ✗ | ✓ | ✗ | ✗ | ✓ |
| Zhou et al. (2016) | ✗ | ✓ | ✗ | ✗ | ✓ |
| Hoffmann et al. (2017) | ✓ | ✗ | ✗ | ✗ | ✓ |

As shown in Table 1, most of the previous studies have used set covering approach to model the railway crew scheduling problem (CSP). In this approach, some trips may be covered by more than one pairing (multiple-covered trips). These trips impose useless transfers to the crew complements, with extra transportation cost.

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## B. Contribution of the Paper

In this study, we propose a new model for the first phase of the railway CSP. The proposed model is called "Transition Reduction", and is capable of simultaneously minimizing both costs of the pairings and the number of transitions. Moreover, a new model for the second phase of the railway CSP is developed to find the optimal solution of the crew assignment. This model attempts to minimize the total cost, including cost of assigning crew complements to the pairings, the fixed cost of employing crew complements and the penalty cost for the short workloads.

## C. Outline

The current paper is organized as follows: In Section 2, the details of the "Transition Reduction" model, focused in this paper, are described. In section 3, the mathematical model, proposed for railway crew assignment is completely elucidated. In section 4, the computational experiments are presented. Finally, the concluding remarks are given at the end to summarize the contributions of this article.

## 2. Transition Reduction: New Model to Find Optimal Pairings

At the first phase of the railway crew scheduling problem, the pairings that constitute the optimal solution (optimal pairings) must be obtained. Most of the previous studies have used "Set Covering Problem (SCP)" to handle this phase. SCP is a prominent combinatorial optimization task which asks to find a collection of subsets to cover all the elements at the minimal cost (Gao et al., 2014). In railway crew scheduling problem, SCP is defined as follows: Let T denote a set of trips, FP a set of all feasible pairings covering the trips. The binary parameter $a_{pt}$ is the input of the problem, which indicates whether trip $t$ is in pairing $p$ or not. This parameter is 1 if pairing p covers trip t and 0 otherwise. $C_p > 0$ is the cost associated with pairing p. Decision variable is $x_p$, where $x_p = 1$ if p is a part of the solution schedule and $x_p = 0$, otherwise. The model aims to find a subset of FP at minimal cost but still covering all trips:

$$Minimize \sum_{p \in FP} C_p x_p \tag{1}$$

$$S.t.$$

$$\sum_{p \in FP} a_{pt} x_p \geq 1 \qquad \forall t \in T \tag{2}$$

$$x_p \in \{0,1\} \qquad \forall p \in \text{FP} \tag{3}$$

Objective function (Eq. 1) is to minimize the total cost of the pairings selected. Constraints (2) guarantee that all trips are covered at least once by the selected pairings. Each of these pairings is associated with the origin depot of its first trip. Hence, each depot is responsible to manage a subset of the pairings.

As mentioned, the constraints of the set covering problem guarantee that each trip is observed at least in one of the selected pairings. Some cases may be occurred in which a specific trip is observed in more than one pairing. For such cases, the multiple-covered trips would lead to impose useless transfers to the crew complements, so that they have to transfer to the home depot of their next trip, either by train as train passengers or even by another transportation alternative like bus, airplane, etc. These extra transfers, are named as "transitions" in this paper. Transitions cause additional costs for the management system, leading to reduce the efficiency of the crew schedules. In order to absolutely avoid such transitions, the "Set Partitioning Problem" (SPP) is modeled, other than the Set Covering Problem (SCP). The mathematical model of SPP is as follows:

$$Minimize \sum_{p \in FP} C_p x_p \tag{4}$$

$$S.t.$$

$$\sum_{p \in FP} a_{pt} x_p = 1 \qquad \forall t \in T \tag{5}$$

$$x_p \in \{0,1\} \qquad \forall p \in \text{FP} \tag{6}$$

The only difference between SPP and SCP is that the constraints are expressed as equality constraints. Consequently, they ensure that all trips are covered exactly once. In other words, each trip is covered by only one pairing. Hence, in case of SPP, the feasible search space is often much constrained than the one associated to SCP. This is the reason why the optimal solution of a set partitioning problem is no better than the optimal one of its corresponding set covering problem. By using SPP model other than SCP model, a considerable amount of good solutions may get lost. Since the probability of the transitions increases by enlarging the scale of the problem, solving the large-scale crew scheduling problems through SPP may lead infeasibility. In other words, for large-scale problems, it is rarely likely to cover all trips without any transition.

In this paper, a new model is proposed, in which the advantages of both previous approaches (SCP and SPP) are maintained. In this model, a value of "Penalty" is considered for any extra transition, in order to minimize the number of the transitions, while keeping the feasible search space as much as possible. We called the model as "Transition Reduction" model, as follows:

$$Minimize \sum_{p \in FP} C_p x_p + \sum_{t \in T} C'_t (y_t - 1) \tag{7}$$

$$S.t.$$

$$\sum_{p \in FP} a_{pt} x_p = y_t \qquad \forall t \in T \tag{8}$$

$$y_t \geq 1 \qquad \forall t \in T \tag{9}$$

$$x_p \in \{0,1\} \qquad \forall p \in \text{FP} \tag{10}$$

$$y_t \geq 0, Integer \quad \forall t \in T \tag{11}$$

Where $C_\text{p}$ and $C'_t$ are the cost of pairing $p$ and the penalty cost imposed for each transition of trip $t$, respectively. The binary parameter $a_{pt}$ indicates whether trip $t$ is in pairing $p$ or not. The binary variable $x_p$ equals 1 if pairing $p$ is selected, and 0 otherwise. The integer variable $y_t$ represents the number of repetitions of trip $t$ in pairings of the optimal solution. A trip observed in more than one of the optimal pairings is named as repeated trip. The objective function (Eq. 7) aims to minimize both cost of the pairings and number of the transitions. According to constraints (8), the frequency of each trip $t$ into the pairings of the solution is equal to $y_t$. Constraints (9) imply that for each trip $t$, variable $y_t$ must be greater than or equal to unity. These constraints ensure that any trip must be covered at least once, in each feasible solution. The relations (10) and (11) show the variables definition.

**3. New Model for Railway Crew Assignment**

At the second phase of the railway crew scheduling problem, the optimal pairings associated to each depot must be assigned to the crew complements of that depot. A crew complement, consisting of at least one but usually two staff complements, covers a sequence of the pairings of the depot. The total crew assignment cost includes the fixed cost of employing crew complements, in addition to the cost of covering the pairings. The crew assignment can be planned for a specified time interval, which can vary between a few days to few weeks. In this paper, the mentioned interval is said to be planning time horizon. In order to handle the railway crew assignment, a new mathematical programming model is proposed. This model is performed independently for each of the home depots. In the model, the number of crew complements must be optimized, rather than being as input data. The solution assigns the crew complements to the selected pairings associated to a unique home depot. In addition, the workload and the fairness of work distribution are concerned. The proposed model constrains the workload amount of each crew complement, within the acceptable limits $\overline{W}$ and $\underline{W}$, which are defined as the maximum allowed duration and minimum required duration for train driving in a working shift, respectively. It is assumed that the violation of $\overline{W}$ is not allowed, whereas the crew workloads which are less than $\underline{W}$ may be accepted by adding penalty costs. Hence, our proposed model accepts the solutions, in which, some crew workloads are less than the minimum required duration ($\underline{W}$). For such solutions, a proportionate penalty cost is imposed to the problem. The notations required to present the model proposed for the second phase of the railway crew scheduling problem are defined in Table 2.

Table 2: Notation of the proposed mathematical model for railway crew assignment

| Notation | | Description |
|---|---|---|
| Sets | $P$ | Pairings set of the depot |
| | $C$ | Crew complements set of the depot |
| Parameters | $NP$ | Number of pairings |
| | $T_P$ | Time duration of the pairing $p$ |
| | $ST_p$ | Starting time of the pairing $p$ |
| | $ET_p$ | Ending time of the pairing $p$ |
| | $\overline{W}$ | Maximum allowed duration for driving in a working shift |
| | $\underline{W}$ | Minimum required duration for driving in a working shift |
| | $C_p^c$ | Assignment cost of the crew complement $c$ to the pairing $p$ |
| | $C_c$ | Fixed cost of employment of the crew complement $c$ |
| | $U_c$ | Workload penalty cost imposed if the workload of crew complement $c$ is less than minimum required duration |
| | $M$ | A large positive integer number |
| Decision variables | $x_p^c$ | (Binary) If the crew complement $c$ is assigned to the pairing $p$, it is equal to 1; otherwise 0. |
| | $y_c$ | (Binary) If the crew complement $c$ is employed, it is equal to 1; otherwise 0. |
| | $z_c$ | (Binary) If the total pairings time of the crew $c$ is less than the minimum required , it is equal to 1; otherwise 0. |

The model is formulated, as follows:

$$Minimize \sum_{c\in C}\sum_{p\in P} C_p^c\, x_p^c + \sum_{c\in C} C_c\, y_c + \sum_{c\in C} U_c\, z_c \quad (12)$$

$S.t.$

$$\sum_{c\in C} x_p^c = 1 \qquad \forall p\in P \quad (13)$$

$$\sum_{c} y_c \le NP \qquad \forall c\in C \quad (14)$$

$$\sum_{p\in P} x_p^c \le NP\,.\,y_c \qquad \forall c\in C \quad (15)$$

$$\sum_{p \in P} x_p^c \, T_p \leq \overline{W} \, . y_c \qquad \forall c \in C \qquad (16)$$

$$\underline{W} \; y_c - M \, z_c \leq \sum_{p \in P} x_p^c T_p \qquad \forall c \in C \qquad (17)$$

$$\underline{W} \; y_c + M \, (1 - z_c) \geq \qquad \forall c \in C \qquad (18)$$

$$x_p^c + x_{p^*}^c \leq y_c \qquad \forall c \in C \; \forall (p, p^*) \,|\, (ST_{p^*} \leq ST_p < ET_{p^*} \qquad (19)$$

$$z_c \leq y_c \qquad \forall c \in C \qquad (20)$$

$$x_p^c \in \{0,1\} \qquad \forall c \in C, \forall p \in P \qquad (21)$$

$$y_c \in \{0,1\} \qquad \forall c \in C \qquad (22)$$

The objective function, presented in Eq. (12), attempts to minimize the total cost, including three parts: the cost of assigning crew complements to the pairings, the fixed cost of employing crew complements and the penalty cost for the short workloads (workload penalty cost). Constraints (13) ensure that each pairing must be assigned to exactly one crew complement. Constraints (14) represent that the total employed crew complements should not exceed the number of pairings (maximum number of employed crew complements). According to constraints (15), if the crew complement $c$ is not employed ( $y_c = 0$), then this complement must not be assigned to any of the pairings. Constraints (16) ensure that the total driving duration of each crew complement does not exceed the maximum allowable period of time ( $\overline{W}$ ). According to constraints (17) and (18), if the total workload of crew complement $c$ is less than the minimum required duration ($\underline{W}$), then binary variable $z_c$ takes the value of unity. The relations (20) and (22) show the variables definition. Note that the value of the penalty costs for the short workloads ($U_c$) depend on the sensitivity of the railway system to the acceptance or rejection of short workloads for each of the crew groups. The more unpleasant the acceptance of short workloads, the higher the value of the penalty costs, and therefore, the more tendency of the model to set the binary variables $Z_c$ equal to zero"

Constraints (19) prevent the assignment of two overlapped pairings to the same crew complement. Figure 1 shows different possible states of two hypothetical pairings p and p* rather to each other. In this figure, $Dur_p$ is the duration of pairing p, which equals the difference between ending time and starting time of pairing p ( $ET_p$ and $ST_p$ , respectively). Also, $RT_p$ is the ending time of the rest duration of pairing $p$ . The mentioned states are as follows:

State 1: $RT_{p^*} < ST_p$

State 2: $ST_{p^*} < RT_p$

State 3: $ST_{p^*} < ST_p < RT_{p^*}$

State 4: $ST_p < ST_{p^*} < RT_p$

If any two pairings are categorized in either state 1 or state 2, then they can simultaneously be assigned to a certain crew complement. On the other hand, the pairings whose relative state lies neither in state 1 nor state 2, are considered as the overlapped pairings and constraints (19) are applied merely for such pairings.

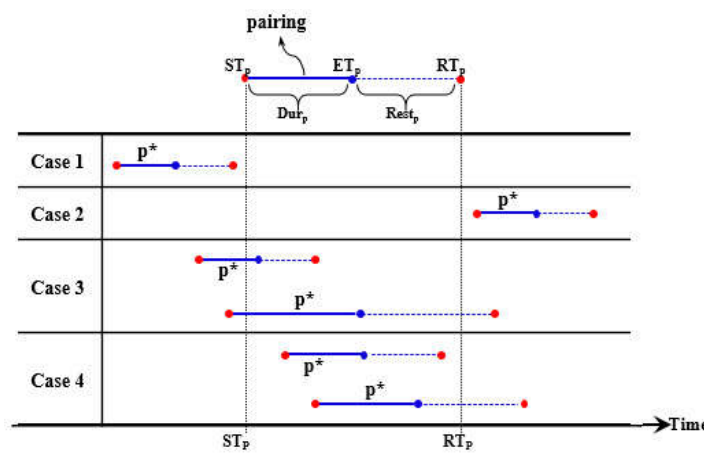


Fig.1. Different possible states of two hypothetical pairings p and p* rather to each other

## 4. Evaluation of proposed models

The evaluation of the proposed models is performed in two parts: at the first part, we have randomly generated several examples with different characteristics. At the second part, the railway network of Islamic Republic of Iran has been investigated as a real-life system. To solve the problems in both parts, a computer with Core 2 CPU at 2.66 GHz and 4 GB RAM was used. The models were solved by CPLEX 12 software which automatically generates optimal solution of each problem.

### 4.1. Evaluation of the models: random examples

For evaluating the models, various random examples were investigated. These examples were solved, by using the proposed "Transition Reduction" model. The characteristics of the examples are shown in Table 3.

Table 3: Characteristics of the random examples, used to evaluate "Transition Reduction" model

| Examples | Number of random trips | Number of depots | Minimum trip duration (hours) | Maximum trip duration (hours) | Minimum Gap between trips (hours) | Maximum duration of a feasible pairing (hours) | Planning horizon (number of workdays) |
|---|---|---|---|---|---|---|---|
| Ex.1 | 500 | 7 | 3 | 15 | 1 | 48 | 7 |
| Ex.2 | 600 | 7 | 3 | 15 | 1 | 48 | 7 |
| Ex.3 | 700 | 7 | 3 | 15 | 1 | 48 | 7 |
| Ex.4 | 800 | 7 | 3 | 15 | 1 | 48 | 7 |
| Ex.5 | 1000 | 7 | 3 | 15 | 1 | 48 | 7 |

As mentioned in Section 2, the objective function of "Transition Reduction" model (Eq. 7) aims to minimize both cost of the pairings and number of the transitions. Parameter $C'_t$ is the transition penalty cost imposed for each transition of trip t. We assume that parameter $C'_t$ is considered N times the trip cost ($C_t$). In other words, $C'_t$ is equal to $N \times C_t$, in which N is called transition penalty coefficient. For instance, if N equals 2, the transition penalty cost for each repetition of a trip is twice the cost of that trip. For each of the examples presented in Table 3, different values of N were examined to analyse the sensitivity of the model to transition penalty cost parameter. In Table 4, the results of the problems corresponding to different values of transition penalty costs, applied for Ex.4 are presented. Similar results can be obtained for other examples.

Table 4: Results of crew scheduling problems with different transition penalty costs, for Ex.4

| Model | Different scenarios | | Num. all feasible Pairings | Num. Optimal pairings | Num. pairings with Repeated trips | Num. repeated trips |
|---|---|---|---|---|---|---|
| SCP | Sc.1 | --- | 20315 | 322 | 86 | 29 |
| Transition Reduction | Sc.2 | $N = 1$ | 20315 | 323 | 69 | 19 |
| | Sc.3 | $N = 2$ | 20315 | 319 | 67 | 19 |
| | Sc.4 | $N = 3$ | 20315 | 324 | 63 | 17 |
| | Sc.5 | $N = 4$ | 20315 | 323 | 57 | 13 |
| | Sc.6 | $N = 5$ | 20315 | 318 | 54 | 11 |
| | Sc.7 | $N = 10$ | 20315 | 312 | 54 | 11 |

The results shown in Table 4, prove that the proposed Transition Reduction model is sensitive to the values of transition penalty coefficients. Regarding the objective function values of different scenarios in Table 4, it is shown that the best (least) value is corresponded to the scenario which applies the unit transition penalty coefficient ($N = 1$). In other words, the most appropriate value for the transition penalty cost of each repetition of a trip, may be equal to the cost of that trip.

Fig.2 illustrates the changes in number of repeated trips in different scenarios, for each of the examples.

According to Table 4 and Fig.2, it is understood that the larger the value of transition penalty coefficient, the fewer both the number of repeated trips and the number of pairing with repeated trips. By increasing the transition penalty coefficient, the rate of reduction in number of repeated trips decreases, but it does not tend to zero. It is worth mentioning that solving each of the above examples (Ex.1 to Ex.5) by Set Partitioning Problem (SPP) model (in which, no repeated trip is acceptable), leads to infeasibility.

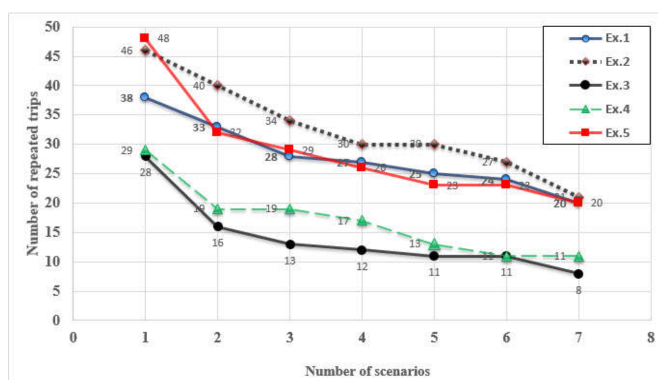


Fig.2. changes of the repetitive trips in various scenarios of the second phase

As noted in Table 3, seven depots (railway stations) are considered in each of the examples. To solve the railway crew assignment in each depot, the model proposed in Section 3 is applied. The model is capable to determine minimum crew complements required to perform all pairings devolved to that depot. For instance, the results of applying the proposed model for crew assignment in one of the depots related to Ex.4, are shown in Table 5. In this table, different values are examined for maximum allowed duration for driving in a working shift ($\overline{W}$) and minimum required duration for driving in a working shift ($\underline{W}$). In this table, the value of transition penalty coefficient is considered unity (N=1).

Table 5: results of proposed crew assignment model for one of the depots related to Ex.4

| | Inputs | | | Outputs | | |
|---|---|---|---|---|---|---|
| Problems | $\underline{W}$ (Days) | $\overline{W}$ (Days) | Number of pairings, devolved to depot | Min crew complements | Objective Function Value | Computation Time (Second) |
| P.1 | 0 | 2 | 59 | 42 | 525404 | 3.4 |
| P.2 | 0 | 2.5 | 59 | 35 | 455404 | 55.8 |
| P.3 | 0 | 3 | 59 | 32 | 425404 | 1.78 |
| P.4 | 0 | 4 | 59 | 32 | 425404 | 2.4 |
| P.5 | 1 | 3 | 59 | 32 | 425404 | 2.9 |
| P.6 | 1.5 | 3 | 59 | 32 | 425404 | 3.7 |
| P.7 | 2 | 3 | 59 | 32 | 6425404 | 42.5 |
| P.8 | 2.5 | 3 | 59 | 32 | 8425404 | 172.6 |

According to Table 5, by increasing the maximum allowed duration for driving in a working shift ($\overline{W}$), the minimum required crew complements decreases. As mentioned earlier, if the total workload of a crew complement is less than the $\underline{W}$, then a workload penalty cost is imposed. That is the case in P.7 and P.8 of Table 5, in which, the workload penalty costs are imposed to the objective function.

### 4.2 Evaluation of the models for a real network: Iranian railways

The proposed models for crew scheduling and crew assignment problems are evaluated, based on the real trip information in Iranian railway network. This network has 18 regions and 27 depots as passenger stations, such that the passenger trips of the network are started from these depots. The information of all scheduled passenger trips of all regions are considered as input of the problem. In Fig. 3, all passenger depots of Iranian railway network are illustrated.

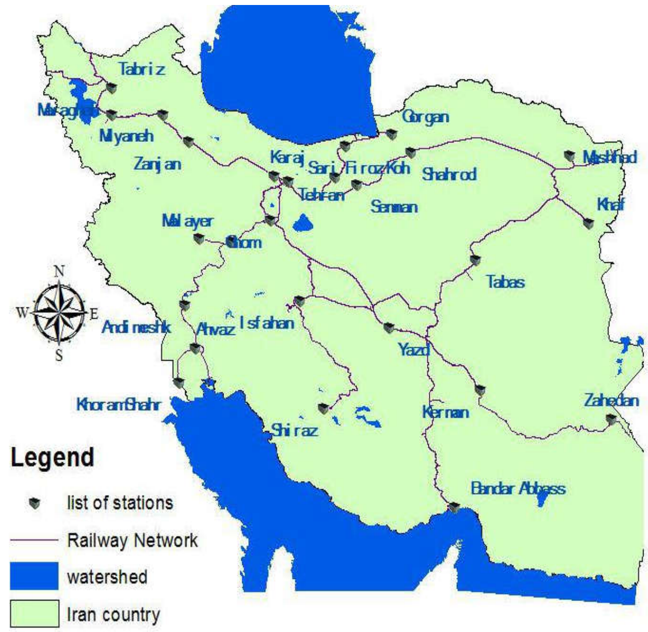


Fig. 3: The passenger depots in Iranian railway network

The input data of the problem includes the timetable of all planned passenger trains of Iranian railways, in 2015. A sample of these data is presented in Table 6.

Table 6: Sample information of passenger train trips in Iranian railways network

| Train No. | Origin | Destination | Start time (Min.) | End time (Min.) | Duration (Min.) | Frequency |
|---|---|---|---|---|---|---|
| 372 | Tehran | Mashhad | 420 | 890 | 470 | Every day |
| 373 | Mashhad | Tehran | 420 | 895 | 475 | Every day |
| 376 | Tehran | Mashhad | 480 | 955 | 475 | Every one day |
| 377 | Mashhad | Tehran | 475 | 950 | 475 | Every one day |
| 580 | Isfahan | Mashhad | 915 | 2000 | 1085 | Every day |
| 581 | Mashhad | Isfahan | 935 | 2015 | 1080 | Every day |
| 520 | Tehran | Isfahan | 1370 | 1770 | 400 | Every one day |
| 521 | Isfahan | Tehran | 1390 | 1795 | 405 | Every one day |

In this research, all of Iranian passenger train trips in a six-day planning horizon are as inputs of the problem. Based on Iranian railway conditions, the maximum time accepted for each pairing and the minimum gap between trips, are 28 hours and 1 hour (60 minutes), respectively. So, the number of all trips in six-day horizon is 1602 trips, which must be assigned by crew implements employed in 27 depots of Iranian railway network.

By regarding the conditions required to constitute trip sequences, 19025 feasible pairings were obtained for the trips of the network. To find the optimal solution of the railway crew scheduling problem, both set covering and Transition Reduction models were used.

In Table 7, the results of the problems corresponding to different values of transition penalty costs, applied for Ex.4 are presented. Similar results can be obtained for other examples.

Table 7: Results of crew scheduling problem, applied for Iranian railway network

| Model | Different scenarios | | Num. all feasible Pairings | Num. Optimal pairings | Num. pairings with repeated trips | Num. repeated trips |
|---|---|---|---|---|---|---|
| SCP | Sc.1 | --- | 19025 | 737 | 104 | 49 |
| Transition Reduction | Sc.2 | $N = 1$ | 19025 | 727 | 76 | 37 |
| | Sc.3 | $N = 2$ | 19025 | 726 | 72 | 35 |
| | Sc.4 | $N = 3$ | 19025 | 724 | 52 | 25 |
| | Sc.5 | $N = 4$ | 19025 | 724 | 52 | 25 |
| | Sc.6 | $N = 5$ | 19025 | 724 | 52 | 25 |
| | Sc.7 | $N = 8$ | 19025 | 727 | 48 | 23 |
| | Sc.8 | $N = 9$ | 19025 | 727 | 48 | 23 |
| | Sc.9 | $N = 10$ | 19025 | 731 | 44 | 21 |
| | Sc.10 | $N = 100$ | 19025 | 732 | 44 | 21 |

The results shown in Table 7 authenticate a significant reduction in both repeated trips and the pairings with repeated trips, when Transition Reduction model is applied. According to these results, the best value for transition penalty cost is corresponded to the scenario which applies the unit transition penalty coefficient ($N = 1$). So, for Iranian railway network, we can consider the transition penalty cost of a trip repetition, equal to the cost of that trip.

By using Transition Reduction model with unit transition penalty coefficient, the proposed model for crew assignment was conducted, separately for different depots of Iranian railway network.

For instance, the results of applying the proposed model for crew assignment in “Yazd” depot are shown in Table 8.

Table 8: results of proposed crew assignment model for “Yazd” depot

| Problems | Inputs | | | Outputs | |
|---|---|---|---|---|---|
| | $\underline{W}$ (Days) | $\overline{W}$ (Days) | Number of pairings, devolved to depot | Minimum crew Complements required | Computation time (Second) |
| P.1 | 0 | 2 | 40 | 20 | 64.4 |
| P.2 | 0 | 2.5 | 40 | 16 | 0.4 |
| P.3 | 0 | 3 | 40 | 16 | 0.2 |
| P.4 | 1 | 2.5 | 40 | 16 | 0.76 |
| P.5 | 1.5 | 2.5 | 40 | 16 | 1.3 |
| P.6 | 2 | 2.5 | 40 | 16 | 1.7 |
| P.7 | 2.5 | 2.5 | 40 | 16 | 12.2 |

The results of Table 8 show that the appropriate values for the upper and lower workloads in Yazd depot, are 2.5 and 2 days, respectively. In Table 9, the results of crew assignment in some main depots of Iranian railway network are presented.

Table 9: results of crew assignment in some main depots of Iranian railway network

| Results \ Depot | Tehran | Mashhad | Shahrood | Isfahan | Tabriz | Bandar Abbas | Yazd |
|---|---|---|---|---|---|---|---|
| Number of pairings, devolved to depot | 224 | 176 | 39 | 15 | 20 | 20 | 40 |
| Minimum crew complements required | 48 | 41 | 11 | 6 | 7 | 7 | 16 |
| Computation time (Second) | 1252 | 186 | 2.6 | 0.4 | 0.36 | 0.85 | 1.7 |

## 5. Conclusions

Railway crew scheduling problem is a substantial part of the railway transportation planning. This problem is based on the train timetable. In this timetable, the specifications of the trips are presented. A sequence of two or more trips is named as a pairing. In a railway crew schedule, all trips of the train timetable must be covered by the feasible pairings. The aim of the railway crew scheduling problem is to find the optimal combination of the pairings with lowest price, and assign them to the crew complements. A small improvement in crew scheduling can lead to huge savings in annual costs of the railway system, which can justify the competitiveness and profitability of the railway system for the operational companies. Therefore, optimizing the railway crew scheduling problem is of interest, in order to reduce the operational costs and increase the profitability of the system. In the optimal solution of the railway crew scheduling problem, each trip must be covered by at least one pairing. However, the multiple-covered trips would lead to impose useless transfers named as “transitions” in this paper. Transitions cause additional costs for the management system, leading to reduce the efficiency of the crew schedules. In this study, we have proposed a new mathematical model for the railway crew scheduling problem, named as “Transition Reduction” model. It is capable of simultaneously minimizing both costs of the pairings and the number of transitions. In this model, transition penalty coefficient is employed to consider the penalty of transitions in the objective function. Moreover, a new mathematical model is proposed to find the optimal solution of the railway crew assignment problem. This model attempts to minimize the total cost, including cost of assigning crew complements to the pairings, the fixed cost of employing crew complements and the penalty cost for the short workloads. To evaluate the proposed models, several random examples, as well as the railway network of Islamic Republic of Iran were investigated. The results of demonstrated the capability of Transition Reduction model to decrease the number of repeated trips. it is understood that the larger the value of transition penalty coefficient, the fewer both the number of repeated trips and the number of pairing with repeated trips. According to these results, the best value for transition penalty cost is corresponded to the scenario which applies the unit transition penalty coefficient. Moreover, the evaluation results of the model proposed for the railway crew assignment shows the ability of this model to minimize the cost of assigning crew complements to the pairings and the fixed cost of crew employment, regarding the short workloads penalty cost.

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