



The progress mechanism of track geometrical irregularity focusing on hanging sleepers

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ABSTRACT

This topic is very traditional and basically has been concerned for a long time. A lot of sophisticated vehicle/track interaction model and track dynamic deterioration model have been developed so far. The author also developed a track settlement progress model comprising a vehicle/track interaction model and a track settlement law. Hanging sleepers are usually caused more or less by the dynamic effect of vehicle/track interaction and has a great impact on the progress of geometrical irregularity of longitudinal track profile. This paper describes that the long term, for seven years, monitoring of track deterioration focusing on actual rail welds and short term monitoring, for three weeks, and dynamic measurements excited by track irregularities set up artificially under the combination with rail surface irregularity considering the typical longitudinal profile of rail welds and track geometrical irregularity considering the typical longitudinal profile of dip joints. Plenty of parameters have influence on the long term monitoring of track settlements and the dynamic measurements of track behaviour, but in this study hanging sleeper was focused on as a key parameter of progressing track geometrical irregularity. Based on the achievements of long term monitoring of track settlements and the dynamic measurements of track behaviour with the focus on rail and/or sleeper displacement and rail bending stress, the impact of hanging sleepers on the geometrical irregularity growth mechanism of track longitudinal profile was discussed and identified to be significant. In addition, the adequacy of track settlement prediction model proposed in this paper was verified to some extent in spite of limited monitoring and measurement conditions.

Keywords: hanging sleepers; track deterioration; track dynamic model; track settlement law

1. Introduction

This topic is very traditional and basically has been concerned for a long time. A lot of sophisticated vehicle/track interaction model and track dynamic deterioration model have been developed so far. The author also developed a track settlement progress model comprising a vehicle/track interaction model and a track settlement law[1]. Hanging sleeper is one of important phenomena to understand the progress mechanism of ballast track geometrical irregularity[2]. In particular, rail joints are a typical structure giving

rise to hanging sleepers, which means local large track settlement takes place due to large dynamic wheel loads excited by traditional fish plate rail joints, and rail welds for continuous welded rails. Track settlement is a key phenomenon for ballast track maintenance, but if track settlement is constant longitudinally, no track geometrical irregularity can be formed. Uneven longitudinal track settlement means track geometrical irregularity, the reason of which is formed by longitudinal variation of track stiffness or the variation of dynamic wheel loads excited by rail surface irregularity such as rail welds, or wheel flat, and so on.

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Then once track settlement is generated at a couple of sleepers, hanging sleepers may be developed due to flexural stiffness of rail. After hanging sleepers occurred, the gap between sleeper and ballast gets large following the increase of dynamic wheel loads up to some extent. Also, the larger the gap between sleeper and ballast becomes, the smaller the interacting force between sleeper and ballast at hanging sleepers and the larger the interacting force between sleeper and ballast at next and/or adjacent sleepers of hanging sleepers, which may suggest the range of hanging sleepers will extend and the track geometrical irregularity will get worse because entropy naturally gets large.

In this paper, track dynamic model to simulate the behaviour of hanging sleeper was verified by experiments to measure track dynamic behaviour particularly with a focus on rail bending stress excited by artificial rail surface irregularity set up at actual track in a revenue railway line. Also, the prediction model of track settlement comprising of track dynamic model and track settlement law was verified and/or checked by the experimental results of long term behaviour of track settlement at some actual track site in a revenue railway line with a focus on typical dynamic behaviour excited at rail welds.

2. Track measurements with a focus on bending stress of rail

2.1 Artificial longitudinal profile of track irregularity

Fig.1 shows the schematic model of artificial track irregularity considering the combination of dip joint and rail surface irregularity based on difference of hardness between rail welded part including heat affected zone and rail matrix material, which is the typical shape of enclosed arc welds.

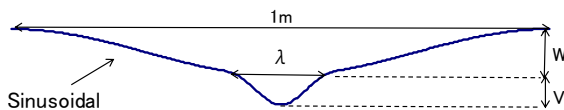


Figure 1. Artificial longitudinal profile of track irregularity

W(mm):Depth of 1m span, V(mm): Depth of span λ (mm)

Also, Table 1 gives the parameters of 15 shapes set up artificially on the running band of rail surface in actual track. Each shape was constructed by some grinders unilaterally in the direction of depth.

Table 1. Technical details of artificial irregularity parameters of V,W, λ (see Fig.1), Unit of W, V and λ : (mm)

Case No.	V	W	λ
1	0.00	0.00	0
2	0.25	0.00	62
3	0.60	0.00	109
4	0.90	0.00	158
5	1.20	0.00	210
6	0.60	0.59	200
7	0.55	0.68	200
8	0.34	0.92	68
9	0.00	1.34	0
10	0.36	1.22	68
11	0.58	1.25	114
12	1.01	1.23	165
13	0.40	1.73	100
14	0.37	1.91	108
15	0.09	2.20	16

Technically speaking, 1m span - irregularity whose parameter of depth is W is basically formed by the geometrical deformation of ballast bed caused by repeated impact loads and generally called as dip joint, and wavelength λ - irregularity whose parameter of depth is V is basically formed by plastic deformation and/or wear based mainly on the difference of hardness between weld material and rail matrix. In this study even artificial 1m span - irregularity was constructed by grinding rail surface longitudinally, which must be pointed out as the difference between artificial irregularity and real irregularity.

2.2 Measurements of rail bending stress

Bending stress of rail was measured with strain gauges installed at the bottom of rail foot where the location of the strain gauge is located in the centre of artificial rail surface irregularity. Also, the two types of railway vehicles, electric express cars and electric locomotives, were focused on. Measured data were analyzed with multiple regression analysis.

Eq. (1) gives the analyzed results associated with electric express cars based on multi-regression analysis. The number of data is 1800, the coefficient of multiple correlation is 0.87 and the standard error is 10.22. The averaged running speed of electric cars is 97.1 km/h and its nominal static wheel load is 58.6kN.

$$Y=0.242U + 49.6V + 18.2W + 22.3 \quad (1)$$

Y: Rail bending stress at the bottom of rail foot (MPa)

U: Running speed of vehicle (km/h)

V, W: Refer to Fig.1 and Table 1

Multiple correlation coefficient: 0.870

Standard error of estimation: 10.22 (MPa)

Number of data: 1800

Also, Eq. (2) gives the analyzed results associated with electric locomotives based on multi-regression analysis. The number of data is 1800, the coefficient of multiple correlation is 0.87 and the standard error is 10.30. The averaged running speed of electric locomotives is 68.9 km/h and its nominal static wheel load is 78.4kN

$$Y=0.292U + 44.0V + 21.0W + 39.0 \quad (2)$$

Y: Rail bending stress at the bottom of rail foot (MPa)

U: Running speed of vehicle (km/h)

V, W: Refer to Fig.1 and Table 1

Multiple correlation coefficient: 0.870

Standard error of estimation: 10.30 (MPa)

Number of data: 1800

Both of analytical results shown as Eqs (1) and (2) seem very good because of 0.87 of multiple correlation coefficient in both cases. Also, roughly speaking, the difference of static wheel loads between electric express cars and electric locomotives can reflect the difference of constant term in Eqs. (1) and Eqs. (2).

3. Track dynamic model

3.1 Track dynamic model and conditions of analysis

Fig. 2 shows a simple track dynamic model adopted in this study [3],[4]. Also, the longitudinal centre of track irregularity set up in the model is the same position of the centre of the two hanging sleepers set up in the model.

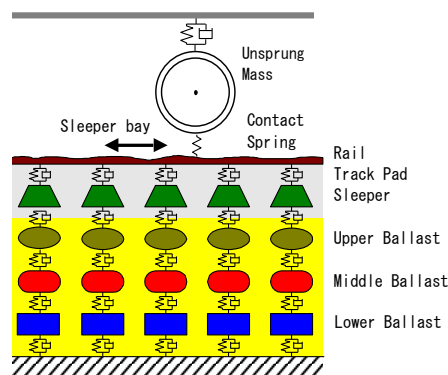


Figure 2. Simple track dynamic model focusing on an un sprung mass of vehicle and three layer – ballast mass

The rail surface irregularities set up in the analysis is the same as the 15 cases given in Table 1. In addition, the gap between sleeper and ballast as the model of hanging sleeper is set as none, 0.5mm, 1.0mm, 1.5mm and 2.0mm. The total number of calculated cases in each railway vehicle of electric express car or electric locomotive is 75; 15 (irregularities) is multiplied by 5 (gaps of hanging sleeper).

3.2 Simulation results

Numerical simulation was carried out using the simple track dynamic model shown in Fig. 2 and track irregularity, whose parameters are shown in Fig. 1 and given in Table 1, including some cases of gap between sleeper and ballast additionally. Running speed of vehicle was set considering the averaged speed of measured data.

Eq. (3) gives the analyzed results associated with electric express cars based on multi-regression analysis. The number of data is 75, the coefficient of multiple correlation is 0.97 and the standard error is 4.39. The

running speed of electric cars was set 100 km/h and its nominal static wheel load is 58.6kN.

$$Y = 49.6V + 18.2W + 13.5L + 22.3 \quad (3)$$

Y: Rail bending stress at the bottom of rail foot (MPa)

V, W: Refer to Fig.1 and Table 1

L: Gap between sleeper and ballast (mm)

Multiple correlation coefficient: 0.97

Standard error of estimation: 4.39 (MPa)

Number of data: 75

Running speed of vehicle: 100(km/h)

Also, Eq. (4) gives the analyzed results associated with electric locomotives based on multi-regression analysis. The number of data is 75, the coefficient of multiple correlation is 0.97 and the standard error is 3.13. The averaged running speed of electric locomotives is 70 km/h and its nominal static wheel load is 78.4kN

$$Y = 39.0V + 7.63W + 14.0L + 53.7 \quad (4)$$

Y: Rail bending stress at the bottom of rail foot (MPa)

V, W: Refer to Fig.1 and Table 1

L: Gap between sleeper and ballast (mm)

Multiple correlation coefficient: 0.98

Standard error of estimation: 3.13 (MPa)

Number of data: 75

Running speed of vehicle: 70(km/h)

Both of analytical results shown as Eqs (3) and (4) seem very good respectively because of 0.97 and 0.98 of multiple correlation coefficient. Both of the coefficients of V and W in the case of electric express cars are larger than those in the case of electric

locomotives, which may suggest the dynamic effect based on running speed. Also, the constant term of electric express cars is much smaller than that of electric locomotives, which may suggest the difference of static wheel loads between electric express cars and electric locomotives.

4. Estimation of the effect of hanging sleeper

Using Eqs (1) to (4), the effect of hanging sleepers based on the gap between sleeper and ballast can be estimated. The gap was estimated in some cases of track irregularities respectively with a focus on the type of railway vehicles.

Table 2 gives the estimated gap under some conditions of electric express car running at the speed of 100km/h using Eqs (1) and (3).

Table 2. Estimated gap of hanging sleepers in the case of electric express cars Y, V, W: see Fig.1, Table 1 and Eq. (1)

Case No.	V: weld irreg. (mm)	W: 1m-span irreg. (mm)	Y: Bending Stress (MPa)	Estimated gap (mm)
1	0.2	0.3	65	0.5
2	0.5	0.5	80	0.8
3	0.5	0.8	99	1.2

Basically track irregularities set in the experiments were progressed, but each parameter did not get larger necessarily because parameter V got smaller than before when parameter W was getting larger in some cases, for example. Also, in such a case, parameter λ was getting larger and finally parameter V disappeared, for example, in the case of No.9 in Table 1. The progress of track irregularity should cause the gap of hanging sleeper to increase totally. From this point of view, estimated gaps in Cases 1 to 3 may suggest the above –mentioned tendency.

Table 3 gives the estimated gap under some conditions of electric locomotives running at the speed of 70km/h using Eqs (2) and (4). Basically the same tendency of the increase of estimated gap discussed in the case of electric express cars was identified in Table 3. But the estimated values of gap in the case of Table 3 are larger than those in the case of Table 2 even though the same tendency as Table 2.

Table 3. Estimated gap of hanging sleepers in the case of electric locomotives Y, V, W: see Fig. 1, Table 1 and Eq. (1)

Case No.	V: weld irreg. (mm)	W: 1m-span irreg. (mm)	Y: Bending Stress (MPa)	Estimated gap (mm)
1	0.2	0.3	77	0.7
2	0.5	0.5	92	1.1
3	0.5	0.8	109	1.4

Track irregularities set in the experiments were progressed, but each parameter did not get larger necessarily because parameter V got smaller than before when parameter W was getting larger in some cases, for example. Also, in such a case, parameter λ was getting larger and finally parameter V disappeared, for example, in the case of No.9 in Table 1. The progress of track irregularity should cause the gap of hanging sleeper to increase totally. From this point of view, estimated gaps in Cases 1 to 3 may suggest the above mentioned tendency. In addition, it may be verified that the simple track dynamic model shown in Fig. 2 may be available for analysing the dynamic behaviour of track excited by hanging sleepers.

5. Long term experiment for track settlement in a revenue railway line

5.1 Layout of long term experiment

Long term measurements have been continued at the track site to understand the long term variation of track settlements and ballast vibrating acceleration tendency with repeated train loads. The track site was set up at a rail weld in a non-electrified single-track. The rail welds were focused on their rail surface irregularities by which large dynamic vehicle/track interaction was excited. The details of track structure at the measurement track site are given in Table 4 Also, some of typical vehicle types running at the track site are in Table 5.

Table 4. Details of track structure at measurement site

Track alignment	Tangent
Rail type (JIS)	50kgN
Structure	rail weld
Sleeper type	PC1-F
Sleeper bay (mm)	620
Ballast thickness (mm)	250
Subgrade type	soil
Annual passing tonnage (ton)	2.3×10^7
Max. running speed(km/h)	130

Rail type and Sleeper type: JIS and Previous Japanese National Railways Standard

Table 5. Typical vehicle types running at measurement site

Express	183, 281, 283 series
Commuter	40, 150 series
Freight (Locomotive)	DD51, DF200 series

Series name: Previous Japanese National Railways Standard

Figs 3 and 4 show the photos of the track site and the pile installation work for measurements at the track site. The pile was installed close to the edge of a reference sleeper which was the nearest sleeper from a rail weld in the longitudinal direction to measure the vertical distance between the base line of pile and a reference sleeper.



Figure 3. Track site for measurements of track settlement

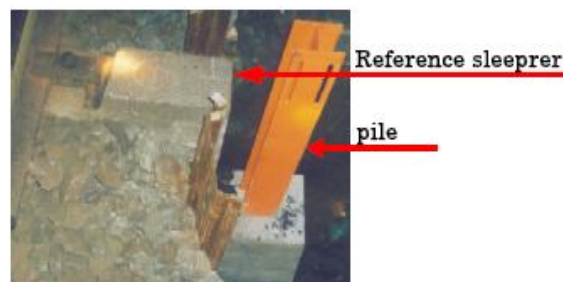


Figure 4. Pile installation work (measuring apparatus)

In the long term measurements, the variation of rail surface irregularity of right rail, the variation of static settlement of the reference sleeper, the variation of dynamic settlement or deflection of the reference sleeper and the variation of ballast vibrating acceleration under the reference sleeper were measured.

Also, in advance of long term measurements of track settlement, some typical track dynamic behaviors at the track site were investigated to identify some

parameters used for the vehicle/track dynamic model. This paper describes the data obtained from seven-year measurements of static and dynamic sleeper settlement, and ballast vibrating acceleration with a pick-up installed into ballast stones and at the place under 100mm from the bottom of the reference sleeper nearest from a rail weld in the longitudinal direction.

On the other hand, the prediction model of track settlement comprising vehicle/track dynamic model and track settlement law was developed [5]-[7] and verified its adequacy using five-year measured data obtained by this long term experiment [8],[9].

5.2 Measured results of long term measurements

Fig. 5 shows the variation of rail surface irregularities of right rail at the initial stage of measurements and the passing tonnage of 156MGT (MGT; Million Gross Ton: 10^6 passing tonnage). In this figure, the variation of rail surface irregularities due to the repeated train loads is much smaller than that based on the findings so far obtained that the progress of the irregularities at rail welds is roughly 0.1 to 0.2mm/100MGT [10],[11].

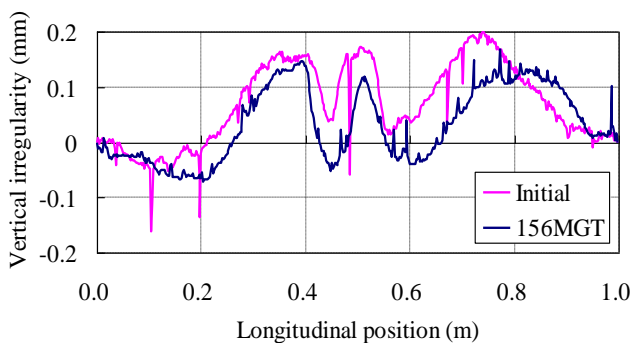


Figure 5. Longitudinal rail surface irregularity

Fig. 6 shows the variations of static sleeper settlement measured at the both ends of right and left rail sides until the accumulated passing tonnage of 156MGT from the start of the measurement. In this figure, the tendency of the progress of static sleeper settlement except some data measured from February in winter to May in spring is very clearly identified that the static sleeper settlement increases with the increase of accumulated passing tonnage.

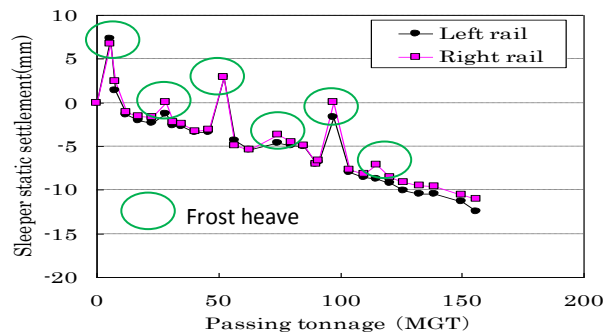


Figure 6. Variation of sleeper static settlement with repeated train loads

On the other hand, the static sleeper settlement was measured at the reference sleeper set up with the focus placed on dynamic behaviour excited by the train running at rail welds. After some train loads passed on the reference sleeper, the sleeper became a hanging sleeper but the static settlement of the reference sleeper can be considered almost the same as the ballast settlement at each sleeper around the reference sleeper at which train loads were not so excited like at rail welds. This is because even if the reference sleeper comes to a hanging sleeper, the bending deflection of the rail hanging a couple of sleepers including the reference sleeper and its next a couple of sleepers at most, not so large number of sleepers, can be negligible.

Also, considering the seasons when the data were obtained may reveal the interesting tendency of frost heaving phenomena because those data were obtained in winter and resumed their tendency in spring. Unfortunately the static sleeper settlement could not be measured in the last February during the long term measurements as data missing shown in the figure. It is not so easy to estimate the effect of frost heave of subgrade on track settlement, but the data obtained in this study can be considered to be caused by the phenomenon of frost heave.

Fig.7 shows the variation of height or magnitude of frost heave with square root of accumulated freezing index which is the special factor of accounting temperature and days below zero temperature to evaluate the degree of frost heave. The height of frost heave is identified to be roughly proportional to the square root of accumulated freezing index. However, the place of weather station to measure temperatures

may not be necessarily close to the track site in this study. The data of temperatures adopted in this study may not be perfectly suitable for evaluating the relation between frost heave and accumulated freezing index. In addition, the data of the last winter were unfortunately not obtained because of snowfall. Also, not so small variation can be identified in Fig. 7. Considering such limited conditions, frost heave surely took place at the place for track site measurements [12].

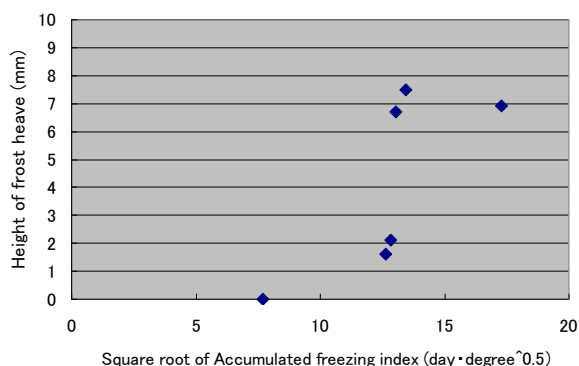


Figure 7. The variation of height or magnitude of frost heave with square root of accumulated freezing index

Fig. 8 shows the variations of dynamic sleeper settlement of both right and left rails until the accumulated passing tonnage of 156MGT from the start of the measurements. In this figure, train loads were classified to commuter trains, express trains and freight trains focusing on vehicle types. In this figure, no hanging sleeper phenomenon was recognised at

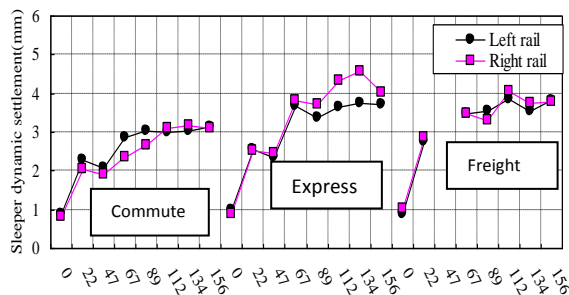


Figure 8. Variation of sleeper dynamic settlement with repeated train loads

the initial stage of measurements because of the dynamic settlement of about 1.0mm, but hanging sleeper can be identified because of the dynamic settlement of more than 1.0mm. The progress of hanging sleepers and the tendency of track settlements

at the rail weld and the mid-span were clarified as the examples of actual track. Then the gap between sleeper and ballast was measured at the passing tonnage of 89MGT. As a method of measuring the gap, the reference sleeper for measurements was unfastened or released from a rail and naturally fallen to the ballast.

Table VI gives the gap measured by the above-mentioned method. In this Table, the average of the gaps at both edges of the sleeper between the sleeper and ballast was about 1.4mm, which was considered to be less than the actual gap of hanging sleeper because 2.6mm (= 4.0mm of dynamic settlement - 1.4mm of the gap) was too large as the elastic displacement of sound ballast which should be almost the same as 1.0mm of dynamic settlement measured just after tamping.

Also, the natural drop must be not enough to estimate the gap of hanging sleeper. At least some load corresponding to the rail seat force estimated from train loads should be applied to the sleeper dropped and placed on ballast. Somehow, the definition of the gap of hanging sleeper being basically ambiguous, the phenomenon of hanging sleeper was recognized and some information of the progress of hanging sleeper was obtained in this study.

Table 6. Gap of hanging sleeper

Place of measurements	Gap (mm)
Sleeper end of right rail side	1.37
Sleeper end of left rail side	1.36
Average	1.37

Fig. 9 shows the variations of ballast vibrating acceleration of both right and left rails until the passing tonnage of 156MGT as well as the dynamic sleeper settlement. In this figure, the very interesting tendency of ballast vibrating acceleration was obtained, which is that the acceleration decreased at the initial stage of the passing tonnage and on the contrary increased after a certain passing tonnage.

Such a tendency can depend on the magnitude of gap as the degree of hanging sleeper, which means the balance between whole track settlement and local track settlement at individual sleepers.

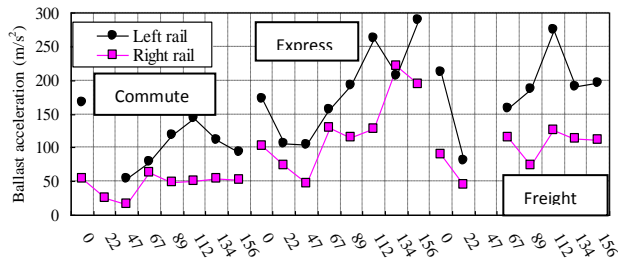


Figure 9. Variation of ballast vibration acceleration with repeated train loads

Also, the cycle of decrease and increase looks repeated during seven-year measurements. This phenomenon may suggest the mechanism of the growth of irregularity of longitudinal level combined with extending hanging sleepers.

5.3 The mechanism of irregularity growth of longitudinal track level

In the long-term measurements of track settlement, rail surface irregularity and static sleeper settlement have been measured as the vertical displacements from the base line of two piles installed very close to both lateral edges of a reference sleeper shown in Fig. 4. Also the dynamic sleeper settlement and the ballast vibrating acceleration have been measured once a year, almost every 23MGT of annual passing tonnage.

From the above discussions, the static sleeper settlement measured in this study reveals ballast settlement caused by repeated train loads which are not so much excited because even if impact loads excited at the rail weld acted the measured sleeper, reference sleeper, so that some gap between sleeper and ballast was generated, the deflection of bending of rail depending hanging sleepers could be ignored. Then ballast settlement grew almost proportion to repeated train loads at least up to 156MGT under vehicle and track conditions adopted in this study. Also, in the case of not so large frost heave, the frost heave had almost no influence on the tendency of ballast settlement. Almost every frost heave settled down along with the tendency of the settlement. Frost heave was confirmed checking the relation between the magnitude of frost heave and accumulated freezing index.

Dynamic sleeper settlement showed its growth rate gradually decreased and looked approaching some

aturated value with some variation. Also, the very interesting tendency of ballast vibrating acceleration was obtained. The acceleration decreased at the initial stage of the passing tonnage and on the contrary increased after a certain passing tonnage. This may suggest the growing mechanism of the magnitude of gap between sleeper and ballast as the degree of hanging sleeper based on the balance between several sleepers within effective length supporting a wheel load. In addition, such a mechanism of extending hanging sleepers must be one of main mechanism of irregularity growth of longitudinal level. Hereby, it is very effective to avoid and repair hanging sleeper from the aspect of reducing maintenance cost and keeping good quality of track.

6. Conclusion

At first, focusing on rail bending fatigue life, track measurements were carried out under the arrangements of artificial track irregularities considering 1m span irregularity like dip joints and short pitch irregularities like rail welds. After that a simple track dynamic model was constructed and used to understand dynamic behavior of track including hanging sleepers. Through such an analysis, it may be verified that the simple track dynamic model shown in Fig. 2 can be available for analyzing the dynamic behavior of track excited by hanging sleepers.

Next, a track site was selected for long term measurements including static sleeper settlements, dynamic sleeper settlements, ballast vibrating acceleration and rail surface irregularity at welds with the focus placed on good subgrade to avoid some disturbance from soft subgrade. The tendency of ballast settlement for seven years was obtained and very interesting behaviour of frost heaving was also clarified even within some limited conditions. Also, the mechanism of growing gap of hanging sleeper between sleeper and ballast interacting with the vicinity of excited sleeper, for instance, near welds was suggested. In addition, such a mechanism of growing hanging sleepers was identified to be one of main mechanisms of growing irregularity of longitudinal track level.

In addition, using the track/ballast settlement data obtained by the above-mentioned long term measurements, the prediction model for ballast settlement combined with the vehicle/track dynamic model and the ballast settlement law proposed in the draft of track design standard in Japan was developed and its adequacy was roughly verified in another paper [13].

Further study on the long term behavior of ballast track will be expected with a focus placed on the interaction between ballast stones, friction and micro fracture of contact points between ballast stones to keep smooth surface of longitudinal track profile for vehicle running as long as possible under the most sufficient maintenance work and minimum maintenance cost [14].

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