



Robust Wide Area Measurement and Control Systems in Railway Smart Network

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ABSTRACT

In this article, a wide area measurement and control system (WAMC) in the railway network is prepared for online monitoring and centralized control of rail vehicles with the aim of creating a smart railway network. The provided wide area measurement and control system increases safety in the movement of rail vehicles. This system can be used to improve the performance of the railway traffic control electric signal system. This system performs three main tasks. First, it performs the task of monitoring the rail network and estimates the current speed and location of all rail vehicles, then identifies dangerous situations, and finally sends the necessary control commands to the control systems for safe movement. For this purpose, we first use the GPS system and Kalman filter to locate railway vehicles and monitor them online. Then, using an algorithm, we identify dangerous positions in the railway network, and finally, the control command is given to the control systems. Moreover, to address the problem of outliers among measurement data, an outlier detection criterion is presented, followed by designing a generalized maximum-likelihood unscented Kalman filter (GM-UKF).

1. Introduction

New railway systems use GPS (Global Positioning System) to track the movement of rail vehicles in real-time. A lot of research has been done and achievements in railway vehicle positioning and signal control based on GPS [1-6]. On the other hand, these systems have problems such as delay, measurement noise, communication channel noise, and cyber-attacks, which are briefly discussed in [7-10] on the effects and methods of facing problems. Wide area measurement and control systems are rapidly gaining ground in engineering sciences such as power systems [11] and [12].

In the UK, the Regulation of Railways Act 1889 [13] introduced a series of requirements on

matters such as the implementation of interlocked block signaling and other safety measures as a direct result of the Armagh rail disaster in that year. In recent years, we have seen great progress in the creation of electrical signal systems in the movement of rail vehicles. In this system, the movement path is divided into time-space blocks. In each of these blocks, only one vehicle is allowed to move, and until the vehicle leaves this block, no other vehicle can enter and move in this block. When the electric signal system becomes unusable due to reasons such as failure, etc., human operators take over the task of traffic control. It is in these situations that sometimes, due to human error, the rule of "movement of only one vehicle in a block" is violated, and two rail vehicles move towards or

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follow each other in a block. This could be disastrous and we are still witnessing such incidents in 2022.

One of the most important safety systems in railways is the Automatic Train Control (ATC) system [14]. The main task of ATC is to control the speed of the train according to the received control inputs. For example, ATC is activated at a certain time and place, and if there is no proper response from the driver, it acts on the emergency brake and stops the car.

The problem of state estimation in engineering science goes back many years. Meanwhile, the Kalman filter is one of the most widely used estimators. The family of Kalman filters are the Kalman filter (KF), the extended Kalman filter (EKF) [15], and the unscented Kalman filter (UKF) [16]. Among the Kalman filters, the KF is suitable for linear systems. The EKF suffers from the approximations due to model linearization, which results in poor state estimation accuracy for highly nonlinear systems such as heavy-loaded power systems. As for the UKF, it improves the accuracy of the EKF, but all the above algorithms are not robust against measurement outliers.

To robustify the WAMC system against measurement outliers (In this article, we have looked at the observation outlier.), we suggest the adaptation of the robust GM-UKF which has been recently presented in [17-20].

In this article, in the first step, by wide area measurement and control system, we design an online monitoring system to monitor the railway network. The purpose of this monitoring is to check the network in detail and obtain the speed and location of all rail vehicles. For online monitoring of the network, the speed and location of rail vehicles are sent to the monitoring center, then after processing and removing the disturbances caused by sending information using the Kalman filter, accurate information is obtained. In the second step, we present an algorithm for identifying railway vehicles at risk. Then the necessary control commands, including reducing or increasing the speed, starting the movement, or stopping completely, are applied to the control system. The combination of measurement systems such as GPS, processors and control systems such as ATC creates a wide area of measurement and control systems in railways. WAMC systems are the basis for smart railway networks. The

existence of smart railway networks ensures the security, stability, and correct operation of railway systems.

The rest of the paper is organized as follows: Section 2 represents the preliminaries on the general structure of smart railway networks. Section 3 describes the automatic train control system, in section 4, the design of a wide area measurement and control system for the railway network has been reviewed, which includes how to connect the system components. Section 5, robust state estimation for railway network and identify dangerous situations and control these situations, and finally in Sections 6 and 7 The verification of the presented method is done and the article is finished.

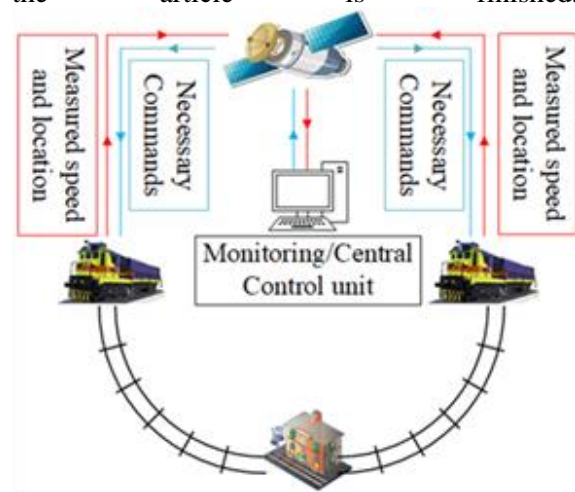


Figure 1. The general approach of the smart railway network.

2. General Structure of Smart Railway Networks

To have a smart railway network, we need to make all the railway components intelligent. Trains and all means of rail transport, stations, traffic systems, and even movement routes must be made smart. Also, there should be an online connection between all these systems or a general processing center should monitor and control all the components of the railway network. Figure 1 shows the general structure of the smart railway network. Measuring devices measure the required information from railway vehicles and railway stations and routes online. Then this information is sent to the central processor through communication channels and optical fibers. In the central processing unit, this information is categorized and checked and combined with train schedules. Dangerous situations such as accidents are predicted. Stops

and movements are managed and to manage all events, control orders and warning orders, and informative information are sent to the components of the railway network. The most important member of the intelligent railway network is the central processing center. This center should receive all the information with less delay and noise and without measuring outliers. Using the received measurement signals and using the GM-UKF algorithm, a centralized state estimation is made to obtain all the network states, and using these states, the state of the network is checked and the necessary commands are sent to the control systems.

3. A Description of Train Automatic Control System (ATC)

The increase in the speed of trains, the increase in the capacity of rail transport networks, the increase in the volume of traffic, the development of stations, the importance of security measures in high-capacity and fast networks, and the inability of humans to reliably control the movement of trains caused the world's advanced railways to increase safety. move to use mechanized equipment under the title of "automatic train control systems". This general title includes warning systems for the driver in times of danger to full control of the locomotive in a driverless train. The automatic control system of the train is running. It is the technological and functional evolution of Automatic Train Protection (ATP). ATC is, in conceptual line, the set of equipment and functionality called ETCS (European Train Control System, S.) in the context of ERTMS (European Rail Train Control System). It is designed to protect the train running against certain constraints and rules like vehicle speed, brake mass percentage; characteristics of the track line section, level indicator, speed side; temporary speed restrictions, slowing down; distancing trains, abnormalities, or details of the signaling systems. It provides onboard continuous information on the maximum permissible speed, taking off the ground continuously, and discontinuously relevant information concerning the freedom of the road, the various physical routes as well as other variable elements. In the event of an overrun of the maximum speed of the train, the ATC conducts adjustment of the speed to bring it back to the allowed values, operand, if necessary, also emergency braking.

4. Designed Wide Area Measurement and Control Systems for Smart Railway Network

In general, the WAMC system is a type of wide system where the system components are located at a large distance from each other. Usually, the components are connected through telecommunication channels or, in some cases, through optical fiber. The main parts of the WAMC system are subsystems, measuring devices, communication channels, central processing units, and control systems. Figure 2 shows how the WAMC system in the railway network works. The most important components of this system are rail vehicles, which are considered as sub-systems in the WAMC system in the railway network. The main purpose of the railway network is to control rail vehicles. In this system, the local control systems receive feedback from the central control unit in addition to the feedback they receive from the local measuring devices. The central control unit receives the speed and location of rail vehicles by local measuring devices that are installed on the vehicles. These measurements are delayed and contaminated with noise after passing through the communication channel or observation outlier.

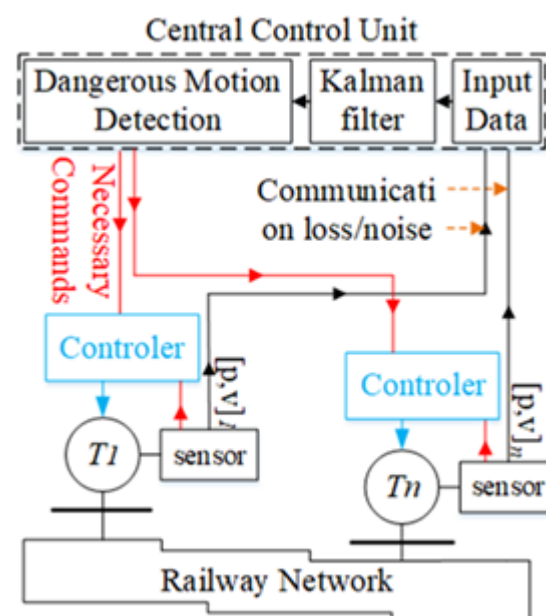


Figure 2. The WAMC system in a railway network.

Using a robust state estimation by GM-UKF and the received measurement, the exact location and speed of vehicles are estimated. Then this information is transferred to a dangerous motion detection algorithm (algorithm 1). Then the appropriate control signals such as reducing or increasing the speed are sent to the local controls

installed on the rail vehicles and the necessary reaction is applied by these controllers.

5. Robust State Estimation for Railway Network

The discrete nonlinear dynamics of the power system can be expressed as follows:

$$\begin{cases} x_k = f(x_{k-1}, u_{k-1}) + w_{k-1} \\ y_k = h(x_k, u_k) + v_k. \end{cases} \quad (1)$$

In (1), x_k is the state vector, u_k is the input vector, and y_k is the output vector. Functions are the state transition and output functions, respectively. The vectors w_k and v_k represent the process and measurement noises, respectively. Having the nonlinear dynamic model of the railway system and the real-time measurements in hand and we can succeed in the robust state estimation using GM-UKF.

5.1. Generalized Maximum-Likelihood-type Unscented Kalman Filter

the robust GM-UKF algorithm [17] and [18] may be applied to achieve the outlier-free estimation of the railway network. Note that the GM-UKF utilizes the same steps of prediction as UKF. To update the predictions, first, it is needed to obtain the uncorrelated representation of the regression problem. The batch mode regression of the UKF is described as

$$\begin{bmatrix} z_k \\ \hat{x}_{k|k-1} \end{bmatrix} = \begin{bmatrix} H_k \\ I \end{bmatrix} x_k + \begin{bmatrix} v_k \\ e_k \end{bmatrix}, \quad (2)$$

in which e_k is the estimation error and the vector $\tilde{v}_k = [v_k \ e_k]^T$ is the total error on measurement and estimation which has the covariance $\tilde{P}_k = \text{diag}(R_k, P_{k|k-1})$

which will be factorized using the Cholesky decomposition method. Multiplying (2) by M_k^{-1} leads to

$$\tilde{z}_k \square M_k^{-1} \begin{bmatrix} z_k \\ \hat{x}_{k|k-1} \end{bmatrix} = T_k x_k + \tilde{v}_k \quad (3)$$

Where $T_k = M_k^{-1} [H_k \ I]^T$ and \tilde{v}_k has the identity covariance I_{n+ny} . Consequently, the estimation equation (3) is uncorrelated.

To solve for the robust Kalman gain, we make use of a Shweppe-type GM-estimator which minimizes an objective function defined as

$$J_k = \sum_{i=1}^{m+n} w_i^2 \rho(r_i), \quad (4)$$

$$r_i = (\tilde{z}_i - a_i^T \hat{x}) / (s w_i).$$

where a_i^T is the i -th row of matrix A_k , $s = 1.4826 b_m \text{median} |z_i - a_i^T \hat{x}|$ is a robust estimation of the scale and parameter b_m is a correction factor. Also $\rho(\cdot)$ is a Huber function given by

$$\rho(r_i) = \begin{cases} 0.5 r_i^2 & -\lambda < r_i < \lambda, \\ \lambda |r_i| - 0.5 \lambda^2 & r_i < -\lambda \text{ or } r_i > \lambda. \end{cases} \quad (5)$$

where the parameter λ determines the shape of the function and is selected to be around 1.5 [19]. Putting the derivative of the objective function equal to zero leads to

$$\frac{\partial J_k}{\partial x_k} = \sum_{i=1}^{m+n} -\frac{w_i a_i}{s} \varphi(r_i) = 0, \quad (6)$$

where $\varphi(\cdot)$ is the derivative of the function concerning? Multiplying and dividing (7) to arrange the equation in matrix form results in

$$T_k^T Q (z_k - T_k x_k) = 0, \quad Q = \text{diag}(q(r_i)), \quad q(r_i) = \varphi(r_i) / r_i. \quad (7)$$

The latter equation can be solved as an iterated recursive least square algorithm as follows:

$$\hat{x}_{k|k}^{(j+1)} = [T_k^T Q^{(j)} T_k]^{-1} T_k Q^{(j)} \tilde{z}_k, \quad (8)$$

where j is the iteration counter and the iterations stop when $\|\hat{x}_{k|k}^{j+1} - \hat{x}_{k|k}^j\| \leq m_{th}$, for example $m_{th}=0.01$. By making use of the influence function of the GM estimator, authors in [17] attain the covariance update rule as

$$P_{k|k} = \frac{E_F[\varphi^2(r_i)]}{(E_F[\partial\varphi/\partial r_i])^2} (T_k^T T_k)^{-1} (T_k^T W_i T_k) (T_k^T T_k)^{-1}, \quad (9)$$

where $W_i = \text{diag}(w_i^2)$.

w_i calculate by the Outlier Detection algorithm.

5.2. Outlier Detection

Outliers are deviant data points among measurements. These may happen in the calculation, data transmission, or processing stages. Outliers among measurement data are likely in communication lines from the sensor to the controller or estimator as a result of bandwidth loss, packet dropouts, or data distortion caused by cyber-attacks.

To address the problem of outlier's state estimation problem, many robust Kalman filters have been proposed in the literature which utilizes the concept of PS.

For a set of sample points $\{z_1, \dots, z_m\}$, the PS values are calculated as ([20])

$$PS_i = \max_{\|v\|=1} \frac{|z_i^T v - \text{med}_j(z_j^T v)|}{1.4826 \text{med}_k \left[|z_k^T v - \text{med}_j(z_j^T v)| \right]}. \quad (10)$$

For $i=1, \dots, m$. Projection statistics are a measure of the sample distance from the center of the observations. Here the sample median is used as the estimator of location (center) and the median-absolute-deviation (or MAD) is used as the estimator of scale. Moreover, the sample median and MAD values are calculated on the direction of all feasible unit vectors. It has been also shown that PS^2 follows the χ_n^2 (Chi-Square) distribution when the data points are coming from a Gaussian distribution [21]. The Projection statistics are applied to the following data points

$$\tilde{Z}_k = [z_{k-1} \quad z_k] \quad (11)$$

where z_{k-1} and z_k are the observation vectors at time instants $k-1$ and k , respectively. Note that we need redundancy in the vector z_k to detect outliers. Moreover, the elements of the vector z_k should be centered around the same value. This is not a problem in frequency estimation for power systems since the value of all frequency measurements is centered around the frequency of one per unit. After calculating PS values for every row of matrix \tilde{Z}_k , outlier weights are calculated as

$$w_i = \min \left(1, (d/PS_i)^2 \right) \quad (12)$$

where d is chosen to be 1.5 to achieve the best statistical performance when noises are following Gaussian distribution.

5.3. Dynamic Model for State Estimation in a railway network

In any state estimation study, state estimation models should be considered. In this study, the

state equations of the railway vehicle are considered as follows:

$$\begin{aligned} P_{k+1} &= P_k + \Delta t V_k + 0.5 a \Delta t^2, \\ V_{k+1} &= V_k + a \Delta t. \end{aligned} \quad (13)$$

Where k is time, P is position, Δt is step time, V is speed and a is acceleration.

5.4. Dangerous Motion Detection Algorithm

The main purpose of this algorithm is to detect dangerous situations in the railway network. After a robust state estimation by the GM-UKF algorithm, the results of this estimation, the speed and location of the railway vehicles are used as the input of this algorithm. In the first stage, it receives the speed and location of all rail vehicles. Thus it has the online status of the rail network. A reliable speed is considered for every type of rail vehicle such as freight or passenger train. Then the online speed of the rail vehicle is compared with the actual speed of the vehicle. If the speed of the vehicle is not within the permissible limit, the movement of the vehicle is identified as dangerous movement, and control commands are applied to the controller of that vehicle according to the type of dangerous situation. We have divided the type of risk situation into three categories: dangerous, very dangerous, and critical. The speed increase system goes out of the circuit in dangerous conditions. In a very dangerous situation, the normal braking system is used to reduce or stop, and in a critical situation, the quick braking system is used to stop immediately.

Algorithm 1: Dangerous Motion Detection

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receive  $(p, v)_i$ ,
for  $i = j = 1, \dots, Nt, Nt = \text{number of train}$ ,
for  $i=1, \dots, Nt$ , if  $V_i > V_{safe}, T_i$  have Dangerous,
Motion.
Detect type of Dangerous Motion,
Send necessary commands to controller  $N_i$ ,
for  $i=1, \dots, Nt$ , if
 $|P_i - P_j| < safe, i \neq j, P_j \neq station$ ,
 $T_i, T_j$  have Dangerous Motion,
Send necessary commands to controller  $N_i$  and  $N_j$ .

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In the second step, the distance of both vehicles is calculated. Of course, vehicles that are safely stopped at the station are not considered. As in the previous step, a safe distance track is considered for both adjacent devices. When this safe distance is lost, a dangerous movement is

detected. And the necessary commands, including slowing down and stopping quickly or moving backward, are sent to the controller of both rail vehicles.

6. Simulation

The wide area measurement and control system in the railway network consists of two main parts: the robust state estimation and the danger point detection algorithm. To verify the robust WAMS system in the railway network, we use a part of the Arak railway network as the studied system. As we can see in Figure 3, this system has 4 stations. The stations on this network are MolkAbad, Arak, Samangan, and Shazand stations respectively. The simulations are performed in 3 scenarios. In the first scenario, we check the robust state estimation algorithm using the GM-UKF algorithm and compare them with the UKF algorithm, and then in the next two scenarios, we check the performance of the danger point detection algorithm. MATLAB software is used for simulations. A system with an AMD CPU and 8 GB of RAM is used.

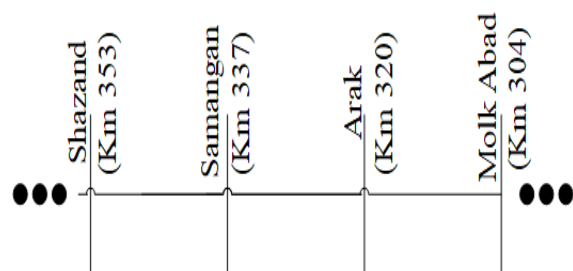


Figure 3. A part of the Arak railway network.

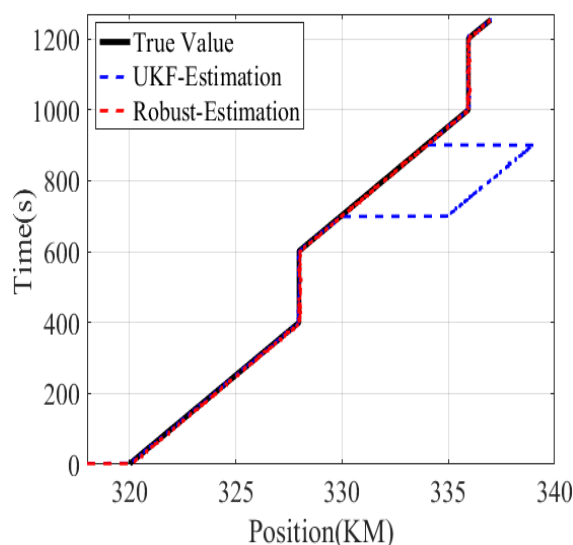


Figure 4. Estimation of the current position of trains, comparing UKF and GM-UKF

6.1. Robust State Estimation in Railway Network

A train starts moving from Arak railway station to Samangan railway station in 0 seconds. In the middle of the way, it stops in 400 to 600 seconds and 1000 to 1200 seconds and then continues to move. It is assumed that the measured location information is contaminated with an outlier in the time interval of 700 to 900 seconds, and in this interval, the measured location is sent to the estimator by more than 5 km. Figure 4 shows the result of comparing the UKF estimation of the train's current position and the robust estimation of the train's current position by GM-UKF. As you can see, the UKF algorithm follows the outlier and makes an incorrect estimate. But the GM-UKF algorithm performs the correct estimation and is robust for outliers.

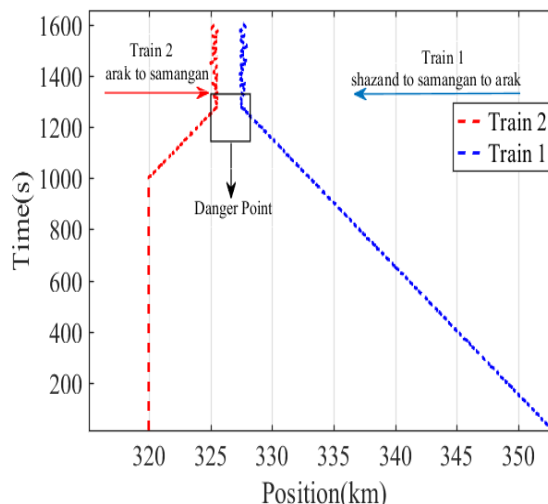


Figure 5. Result of performance of the presented algorithm at the time of an accident

6.2. Checking the performance of the presented algorithm at the time of an accident

In this scenario, we will simulate an accident in the railway network and check the performance of the presented algorithm. It is assumed that the electrical signal system for traffic control has a technical failure and traffic control is done by human operators. The first train starts moving from Shazand station to Arak station in 0 seconds. At 800 seconds, it reached Samangan station and, according to the movement command by the human operator, it continued to move towards Arak station. The second train moved from Arak station to

Samangan station in 1000 seconds due to human error. Two trains are moving towards each other and the probability of an accident is 100%. The current position and current velocity of the trains are estimated by the presented algorithm. After the distance between the two trains is less than the safe value (assumed 7 km), the command of negative acceleration and deceleration is applied to the controllers. And by reducing the distance of two trains from the danger value (assumed 3 km), the command of emergency braking and full stopping is applied to the trains at the moment of 1250 seconds. The trains brake completely in 1290 seconds and stop completely before the collision. You can see the presented results in Figure 5.

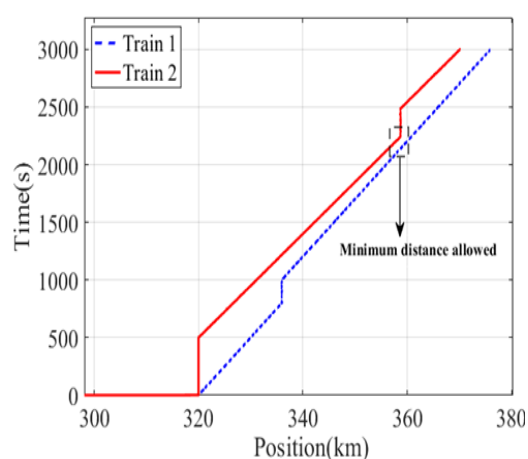


Figure 6. The movement of two trains following each other (Smart tracking), Instantaneous graph of the position of trains

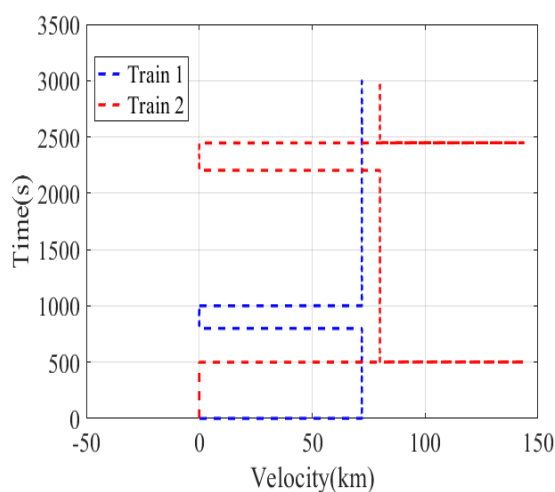


Figure 7. The movement of two trains following each other (Smart tracking), Instantaneous graph of the velocity of trains

6.3. Smart tracking in the railway network

It is assumed that train 1 starts moving from Arak station to Samangan station and then Shazand at 0 seconds. Train 2 departs from Arak station 500 seconds after train 1. As shown in Figures 6 and 7, train 1 stops in 750 to 1000 seconds and then continues its movement. As a result, the safe distance between two trains decreases and the presented algorithm comes into action. At the moment of 2100 seconds, the distance between the two trains decreases from the safe point, and as a result, the negative acceleration and deceleration command is applied to train 2, while no command is applied to train 1 to decelerate. At 2200 seconds, due to the reduction of the distance of train 2 from the critical point (assumed 2 km), train 2 brakes completely and stops, and after 300 seconds, due to the movement of train 1, a safe distance is created and train 2 starts moving again.

7. Conclusion

A wide area measurement and control system, which is the most important step in creating a smart railway network. centralized state estimation in this system is very important. This estimation must be robust to the outlier. GM-UKF algorithm is robust to outliers. The next step is to detect dangerous situations and manage these situations, which is possible with the presented algorithm. In general, despite its simplicity, this system has very high efficiency and is capable of controlling the traffic of the railway network. Also, because a central processing system issues the necessary commands, as a result, the development of this system is simple and less expensive. It is suggested to integrate this system with the rail traffic system to develop its work and stoppages such as refueling, oiling, and repairs should be predicted.

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