



Practical Modeling of Existing Trains in the Shiraz Metro Using Neural Networks and PID-Fuzzy Controller for ATO System Implementation

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ABSTRACT

Transportation optimization is considered one of the main aspects of development in smart cities nowadays. Urban rail transport systems, as one of the key elements of passenger movement in large cities, have witnessed significant growth and advancement in utilizing modern knowledge to provide services to passengers over the years. The existence of sufficient infrastructure and the importance of better service provision to passengers have led these systems to always stay at the forefront of recent advancements in data analytics and artificial intelligence. Automatic Train Operation (ATO) is one of the infrastructures currently being implemented in urban trains worldwide, and its quality heavily relies on the accurate analysis of train status and signaling systems. This article aims to extract practical data from the movement of trains in the Shiraz metro, model it using a neural network, and then propose a new methodology for simulating and implementing the ATO system in urban trains through the design of a Fuzzy-PID controller.

1. Introduction

Public transportation is recognized as one of the key indicators of human society's development. The development of urban structures, considering population growth on the one hand and the increase in environmental pollution on the other, has made life without public transportation systems extremely challenging. This has prompted researchers in this field to constantly seek innovative solutions to enhance the efficiency of these systems.

These solutions, both in terms of creating innovative methods and optimizing existing systems, are not feasible without modeling, considering the high cost involved in this field. Providing models that can describe the behavior of these systems is both highly beneficial and challenging. In this regard, urban rail transportation systems have been at the center of

attention due to their extensive scope and importance. High energy consumption, the need to ensure passenger comfort [1], and the high level of reliability required by these systems have motivated researchers to constantly seek methods for analyzing their behavior. Modeling trains and implementing appropriate control systems are among the methods that have always been of interest. This approach can prevent issues before they occur. Therefore, various methods have been introduced in different articles. For example, in reference [2], an attempt has been made to introduce equations that influence vehicle movement. These equations have then been used for modeling transportation systems. In reference [3], a mathematical model has been utilized for modeling the existing trains in the urban transportation system. Despite the complexities involved, using mathematical equations governing the system has made the analysis highly dependent on the accuracy of these

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equations. However, in some cases, due to the approximations applied, the analysis may be unable to capture certain real-world conditions. In articles [4-7], studies have been conducted on the Madrid Metro, focusing on the development of a temporal model for regulating and optimizing energy consumption in the ATO system. Similar research has been carried out on high-speed trains in references [8-10] following the same approach. While these articles have made valuable strides in modeling train behavior, these methods also have limitations such as time-consuming processes, complexity, modeling without feedback, and heavy reliance on parameters that are challenging to determine practically. For instance, physical resistances in train movement or the rotational mass of the train are among the parameters that need to be input. Significant research has also been conducted in the field of train modeling using artificial intelligence. However, most of them focused either on centralized maintenance processes [11-12] or did not consider control systems [13-15]. In reference [16], an attempt was made to create a model of a single train using a neural network and a small amount of data (12 data points). In this reference, only speed and acceleration were used as inputs, neglecting the impact of other influential parameters such as weight as well as environmental factors like gradient and curvature. Although the author, in their subsequent paper [17] in 2019, attempted to address this limitation and increase the required amount of data for neural network modeling, the study was still conducted on a single train, and no practical model for the power supply network was presented.

In reference [18], an interval of 145 seconds was attempted to be modeled using a feed-forward neural network model. This was achieved by employing a mathematical model of the train. However, a notable point is that the characteristics of speed, current, and distance were considered uniform between all stations, resulting in the train exhibiting the same behavior throughout the entire route in practice. In references [19 and 20], the data related to the Beijing Metro were used to investigate their impact with and without regenerative braking. However, in these references, a model for the train itself was not considered, and only the initial data was used for this purpose.

In this article, an attempt has been made to cover aspects that have not been adequately addressed in previous references. These include the online use of practical data, the utilization of a closed-loop model that can control the train's behavior according to the desired output, and the application of fuzzy logic, which is highly similar to human behavior and serves as a suitable simulator for modeling the behavior of operators in response to various parameters.

2. Approach

Based on the objective of creating a train model using practical data, the following steps have been performed in order:

2.1. Data Collection: The first step is to gather practical data, which is achieved by installing highly accurate data loggers on the existing trains in the line. This data is then compiled into a valuable library.

2.2. Data Preprocessing: Considering the nature of the train and the parameters that affect the train's traction, the desired data is extracted from the raw primary data. This involves separating relevant information from the available raw data.

2.3. Neural Network Modeling: A neural network model was developed using the collected data. The neural network would be trained to learn the patterns and relationships within the data, enabling it to simulate the train's behavior accurately.

2.4. Fuzzy Logic Controller Design: To enhance the simulation and implementation of the Automatic Train Operation (ATO) system, a Fuzzy-PID controller was designed. The fuzzy logic approach allowed for handling uncertainties and imprecise inputs, ensuring a robust control mechanism.

2.5. Simulation and Implementation: The designed model, along with the Fuzzy-PID controller, was simulated and implemented to evaluate its performance. This step involved testing the model's ability to replicate real-world train movements and assessing the effectiveness of the ATO system.

By following these steps, the article aimed to create a train model using practical data, incorporating neural network modeling, and

using a Fuzzy-PID controller to simulate and implement an ATO system for urban trains.

3. Data Collection

The required data in this article is collected automatically and online. This process is performed by a highly accurate train data logger with a sampling rate of 100 milliseconds. The extracted raw data includes all measurable electrical and mechanical characteristics of the train. To better analyze the train behavior and consider factors related to its position, the raw data is classified into separate libraries corresponding to the routes between two stations. This process essentially determines the behavior of trains moving on these routes. Figure 1 illustrates a sample of this data (speed and current).

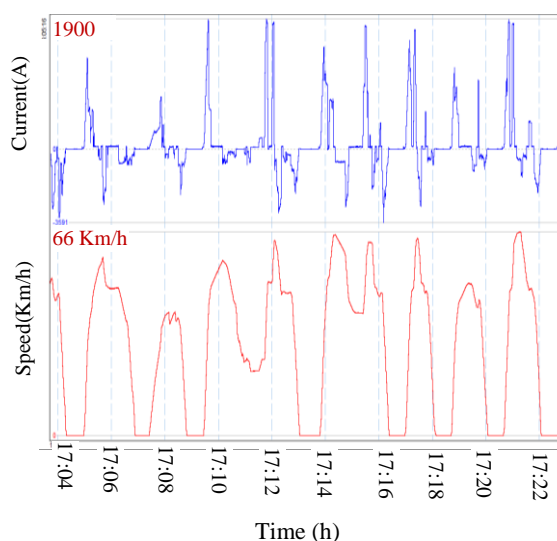


Figure 1. A sample of speed and current data recorded by the train data logger.

4. Analysis, Extraction of Raw Data, and Extraction of Required Information for Train Modeling

The parameters that affect the train behavior are the same parameters that the train must overcome in order to move. In Figure 2, some of these parameters can be observed briefly. Therefore, based on the above equations, the parameters considered as inputs for the neural network are:

1. Train position on the track;
2. Train weight;
3. Instantaneous train speed;

4. Instantaneous train acceleration;
5. Brake request level;
6. Traction request level; and
7. Track gradient and incline.

The final extracted model should be able to model the train behavior using data rows 1 to 7. This behavior can be the instantaneous train velocity or even the power consumption. In Figure 3, traction curves, velocity curves, and acceleration curves for some train movements between two stations can be observed. The dependency of train traction on the mentioned parameters can be observed in Figure 3. The repeatability of this behavior and the availability of a large amount of data make neural networks a suitable choice for modeling. Among various modeling methods for physical systems, neural networks are utilized due to their ability to capture complex patterns and relationships in the data.

5. Modeling the Train Using Neural Networks

In this section, we focus on modeling the train using a neural network.

5.1- Determining the Neural Network Structure

Neural networks are categorized as soft modeling methods and are used for modeling complex systems and nonlinear data classification. This method is based on the modeling of intelligent neural systems found in nature. Just like natural neural networks, the effectiveness of artificial neural networks lies in their trainability. This allows them to be used for modeling structures that cannot be easily modeled using analytical methods. Neurons play a central role in neural networks. The arrangement of these neurons and their interconnections can determine their effectiveness. Neurons are organized into different layers in a neural network. These layers are generally divided into three categories:

- Input layer;
- Hidden layer(s); and
- Output layer.

The number of neurons in the input layer is equal to the number of inputs, and the number of neurons in the output layer depends on the

F_{wt} = Final resistance
 F_R = Wheel friction resistance
 F_D = Aerodynamic resistance
 F_B = Resistance against train acceleration
 F_{st} = Resistance due to slope
 ρ = Air density
 c_w = Aerodynamic resistance coefficient
 A = Cross-sectional area
 v = Velocity
 k_r = Rolling resistance coefficient
 m = Mass
 g = Gravitational constant
 α = Angle relative to the horizon
 k_m = Acceleration resistance
 a = Acceleration

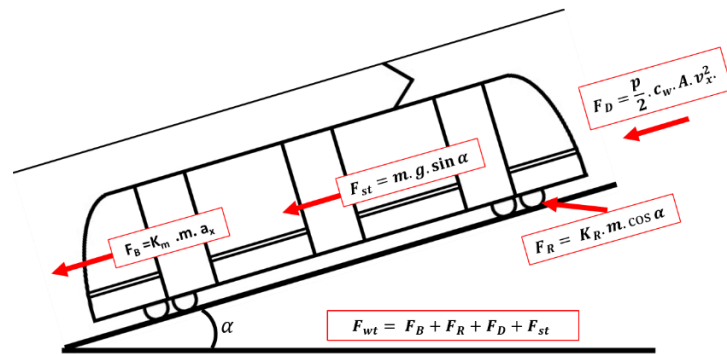


Figure 2. Equations Governing Train Motion.

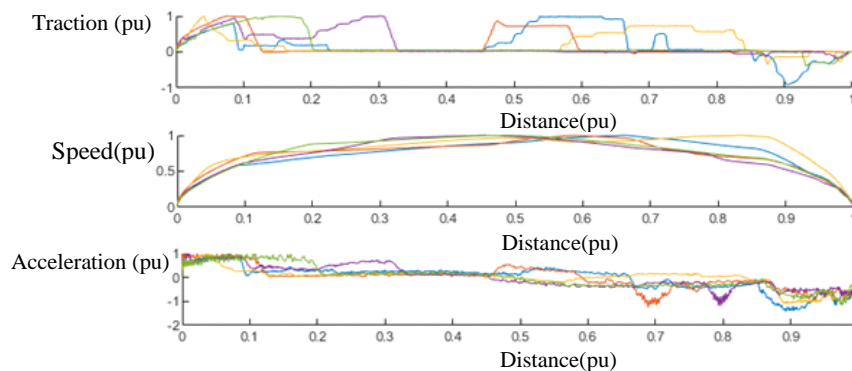


Figure 3. An example of extracted data from train movements.

desired outputs. The selection of the number of neurons in the hidden layer is crucial in modeling using neural networks. Too many neurons can lead to network complexity and overfitting, while too few neurons can result in increased errors. Different references suggest various methods for determining the number of neurons in the hidden layer. For example, in reference [21], it is recommended to have slightly more neurons in the hidden layer than the number of neurons in the input layer. In reference [22], it is proposed to gradually increase the number of neurons in the hidden layer until the system error starts to increase. The number of neurons that produces the minimum error is chosen as the number of neurons in the hidden layer. Other references, including reference [23], determine this number using the formula $\sqrt{(N \times M)}$, where N is the number of neurons in the input layer and M is the number

of neurons in the output layer. Other approximations, such as using 75% to 300% of the input neurons, are also mentioned in this reference. In this article, the number of neurons in the hidden layer is determined using the proposed formula in reference [24], which utilizes validation errors. This method is slightly similar to the one in reference [22]. In this reference, the error of the training data is used. The advantage of using a validation error instead of a training error is that as the number of neurons in the hidden layer increases, the training error always decreases due to overfitting, but this phenomenon leads to an increase in the validation error. Therefore, overfitting cannot cause errors in determining the number of neurons in the hidden layer with this approach. This phenomenon can be clearly observed in Figure 4.

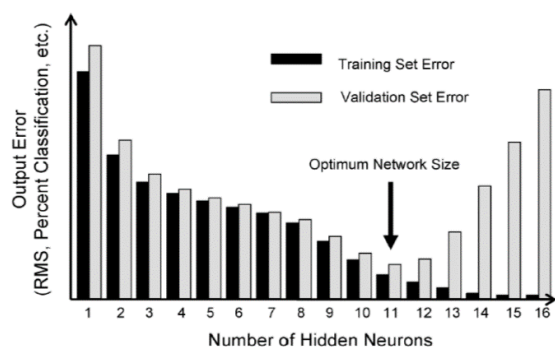


Figure 4. Determining the number of neurons in the hidden layer using validation error [11].

The algorithm used in this article to find the optimal number of hidden layer neurons is the genetic algorithm. The reason for using the genetic algorithm is its advantage in escaping local minima. Figure 5 shows the result of determining the optimal number of hidden layer neurons using the genetic algorithm.

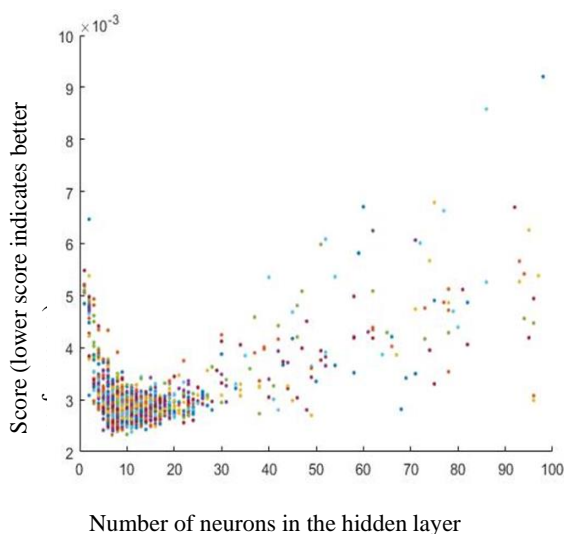


Figure 5. Result of the genetic algorithm for determining the optimal number of neurons in the hidden layer.

The optimal number of neurons obtained by the genetic algorithm is 6, which can be clearly observed. This number is consistent with all the other criteria discussed previously. This model has been previously developed in reference [25] by some authors of this article.

5.2. Training the neural network

In order to train the neural network, data extracted from the trains operating on the Shiraz

Metro Line 1 has been utilized. This line is 24 kilometers long and consists of eight traction substations and 21 stations. The trains on this line are powered by DC (Direct Current), and the voltage used for their supply is 1500 volts. Figure 6 illustrates the schematic of this line.



Figure 6. The Shiraz Metro line, from which modeling data has been extracted.

The initial models in this article have been trained using one thousand inputs for each section (distance between two stations). These data correspond to 50,000 kilometers of train traffic on the Shiraz metro line. The data has been preprocessed for input to the neural network. Seventy-five percent of the data has been used as training data, 15% as test data, and 15% as validation data.

6. Selection of Control System for ATO Modeling

One of the specifications of an ATO (Automatic Train Operation) system is determining the train's behavior based on predefined patterns and relevant indicators. One of the most important indicators is the speed profile. Although other indicators such as energy consumption and passenger comfort are also involved [1], it is evident that both of these indicators can be considered in the content of the speed curve. Therefore, modeling a system that can adhere to this curve can address the requirements of a complete ATO system. In this article, the speed component is considered the controlled component, and the traction-brake component is used as the input to the model. Figure 7 illustrates the schematic of this model.

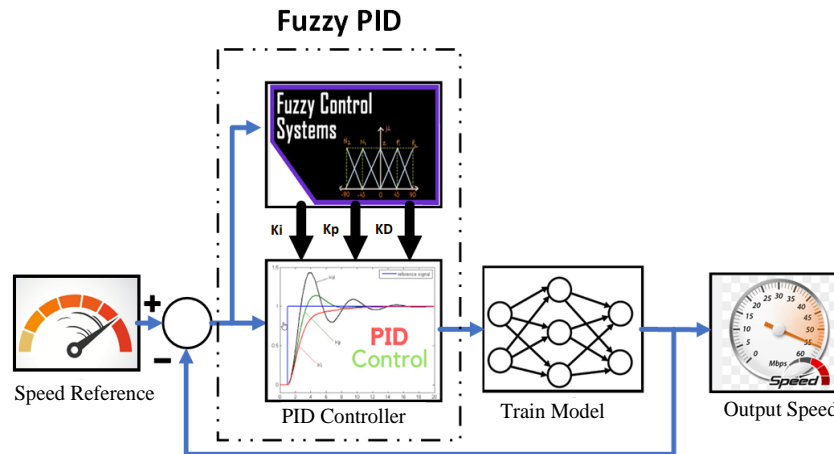


Figure 7. Schematic block diagram of train speed control system.

6.1. Tuning of Fuzzy PID Controller

One of the advantages of a fuzzy-PID controller may be that the PID controller coefficients, which were previously discussed, vary according to changing conditions and have dynamic behavior to reduce error. The standard form of the transfer function of a PID controller can be observed in Equation (1).

$$G_c(s) = K_p + \frac{K_I}{s} + K_d s \quad (1)$$

Actually, fuzzy systems perform these changes using the error and its derivative. To tune the controller, its fuzzy components should be designed. To facilitate the computations, let's first rewrite the main equation of the controller, described in Equation (1), as follows:

$$G_c(s) = K_p \left(1 + \frac{1}{T_i \cdot s} + T_d \cdot s \right) \quad (2)$$

Based on this formula, the coefficients of the fuzzy controller can be defined as follows:

$$K_i = \frac{K_p}{T_i} \quad (3)$$

$$K_d = K_p \cdot T_d \quad (4)$$

Given the above equations, it is possible to express one of the variables as a fundamental PID controller based on the other variables. To achieve this, a new variable called α is defined as follows:

$$\alpha = \frac{T_i}{T_d} \quad (5)$$

Therefore, considering Equations 3 to 5, the integral controller can be written as follows:

$$K_i = \frac{K_p^2}{\alpha \cdot K_d} \quad (6)$$

However, the advantage of using fuzzy logic in determining the parameters of the PID controller is that in this logic, the parameters are not absolute values and can be defined as a range of numbers. For this reason, the parameters related to fuzzy logic are defined as follows [26]:

$$K'_p = \frac{K_p - K_p^{\min}}{K_p^{\max} - K_p^{\min}} \in [0,1] \quad (7)$$

$$K'_d = \frac{K_d - K_d^{\min}}{K_d^{\max} - K_d^{\min}} \in [0,1] \quad (8)$$

Equations 7 and 8 actually indicate that the PID control gains are uncertain and normalized in the range of [0-1]. In this case, determining the range of the variable α is crucial. This variable can be obtained using the Ziegler-Nichols method, approximately around the value of 4 [26]. However, as mentioned earlier, when defining variables in fuzzy logic, they should be defined within a certain range. Therefore, this variable is also defined within the corresponding range as follows:

$$\alpha \in 2 \leq \alpha \leq 5 \quad (9)$$

Once the fuzzy system variables are determined, the main step in designing this system is to write the rules of the system. The first step for this purpose is the distribution of input data, which in this case refers to the error and its derivative. The most common approach used in fuzzy logic algorithms is to use triangular membership functions for this purpose. The centers of these triangles are placed at zero for maintaining symmetry, and their two sides extend to 1- and 1, respectively, to better distribute the data across the seven curves. Additionally, for generating the output, two curves named "Small" and "Big" are used, both of which actually determine the gain values. The transition from the "Small" to the "Big" value should be done in the smoothest possible manner. In Figure 8, these two curves can be observed.

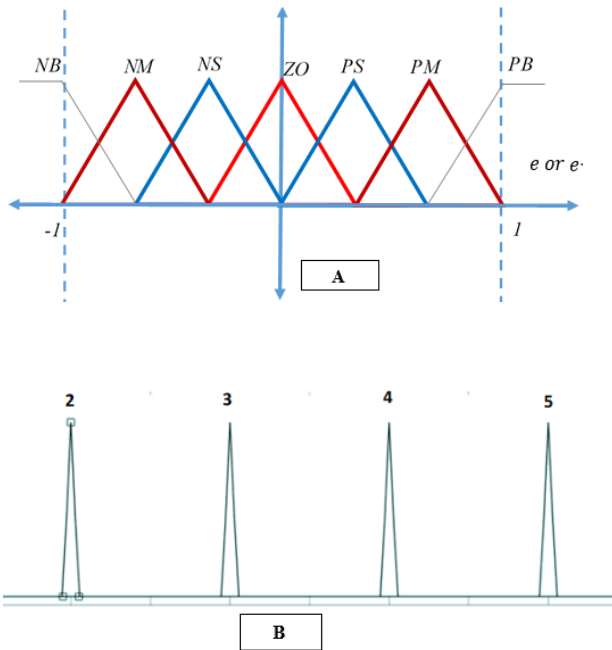


Figure 8. (a) Distribution of fuzzy inputs in the system as triangular fuzzy functions. (b) Output curve for α .

Based on the input function definition, the standard format of fuzzy rules can now be written. The main format of these rules has been designed according to reference [26] as follows:

If Input(1) is A and Input(2) is B ,
Then Output is C

Here, instead of Input(1) and Input(2), there are variables such as e^- and e , representing the error and its derivative, respectively. Instead of

A and B, any of the curves shown in Figure 9 can be used. Instead of Output, using any of the PID controller variables defined in Equations 7 to 9 is recommended, and instead of C, the ranges within which these input variables should vary are utilized. It is worth noting that due to the normalization of the specified ranges, there is practically no concern about the upper and lower limits of these ranges.

In order to allocate fuzzy rules, considering the references [Fuzzy Gain Scheduling of PID Controllers] and [Ziegler-Nichols], the concept of rules is determined with respect to the standard step response of the system. In Figure 9, a standard step response is depicted.

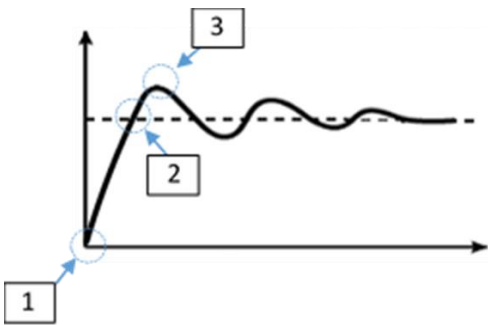


Figure 9. represents the standard step response of a control system to a step input.

Typically, for achieving a good step response, a PID controller should have the following gains in the regions mentioned in Figure 10:

Table 1. Standard Gains for an Ideal PID Controller

Gains	Zone 1	Zone 2	Zone 3
$ K_p $	High	Low	High
$ K_i $	High	Low	High
$ K_d $	Low	High	Low

The same table can be written for the specified gains in Equations 3 to 5 as Table 2.

Table 2. Standard Gains for an Ideal PID Controller

Gains	Zone 1	Zone 2	Zone 3
$ K_p' $	High	Low	High
α	Low	High	Low
$ K_d' $	Low	High	Low

Based on Tables 1 and 2 and Figure 8, the rules of the fuzzy decision-making system can be written as follows:

The phrases included in Tables 3 and 4 indicate the values "low" and "high." Therefore, it can be inferred that the output curves corresponding to K_d and K_p gains will have two curves: "high" and "low." Moreover, one of the best curves that

can oscillate smoothly between these two values might be defined as Figure 10.

Table 3. Fuzzy System Rules for Determining K_p

	e'						
	NB	NM	NS	ZO	PS	PM	PB
e	NB	High	High	High	High	High	High
	NM	Low	High	High	High	High	Low
	NS	Low	Low	High	High	Low	Low
	ZO	Low	Low	Low	High	Low	Low
	PS	Low	Low	High	High	High	Low
	PM	Low	High	High	High	High	Low
	PB	High	High	High	High	High	High

Table 4. Fuzzy System Rules for Determining K_d

	e'						
	NB	NM	NS	ZO	PS	PM	PB
e	NB	Low	Low	Low	Low	Low	Low
	NM	High	High	Low	Low	Low	High
	NS	High	High	Low	High	High	High
	ZO	High	High	High	High	High	High
	PS	High	High	Low	High	High	High
	PM	High	High	Low	Low	Low	High
	PB	Low	Low	Low	Low	Low	Low

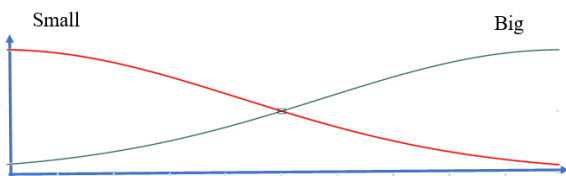


Figure 10. Output Transformation Curve for K_p and K_d .

As mentioned in Equation 9, the parameter α takes values between 2 and 5 [26]. Therefore, considering Figure 10 and Table 2, this parameter can be determined as follows:

Table 5. Fuzzy System Rules for Determining α

	e'						
	NB	NM	NS	ZO	PS	PM	PB
e	NB	2	2	2	2	2	2
	NM	3	3	2	2	2	3
	NS	4	3	3	2	3	4
	ZO	5	4	3	3	4	5
	PS	4	3	3	2	2	4
	PM	3	3	2	2	2	3
	PB	2	2	2	2	2	2

The other parameters that need to be determined for fuzzy system tuning are the boundary limits referred to in Equations 7 and 8. For this purpose, the Ziegler-Nichols method and references [26] and [27] can be utilized. Since these references provide detailed information on these limits and how to calculate

them, further discussion regarding this matter is not included in this article. It is only emphasized that these limits are set based on the capabilities and advantages of the fuzzy controller system, which can vary between a classical PID controller and a controller with overshoot. Accordingly, the boundary values are determined as follows:

Table 6. PID Controller Gain Threshold Values.

Gains	Maximum	Minimum
k_d	7.1	3.8
k_p	1.64	0.88

Therefore, to restore the gains from the normalized state to the applicable state for the controller, the following equations and the numbers mentioned in Table 6 should be used:

$$K_p = K_p^{min} + (K_p^{max} - K_p^{min}) \cdot K'_p \quad (9)$$

$$K_d = K_d^{min} + (K_d^{max} - K_d^{min}) \cdot K'_d \quad (10)$$

$$K_i = \frac{K_p^2}{\alpha \cdot K_d} \quad (11)$$

7. Implementation of Fuzzy PID Controller for Neural Network Train Model Speed Control

To implement this system and determine its parameters, first the parameters of a simple PID controller are determined using the Ziegler-Nichols method. The practical train model, which was discussed in previous sections focusing on its creation, is considered the system that needs to be controlled by this controller. The simulation of this section was performed using MATLAB Simulink software. Figure 11 illustrates this model.

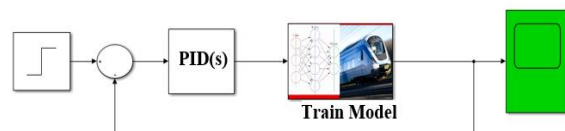


Figure 11. Block diagram of the train model simulation with a PID controller.

As indicated in reference [26], in order to determine the gains of a PID system, a parameter called K_u is required. This parameter is used to achieve a balance with a certain value of K_p in a

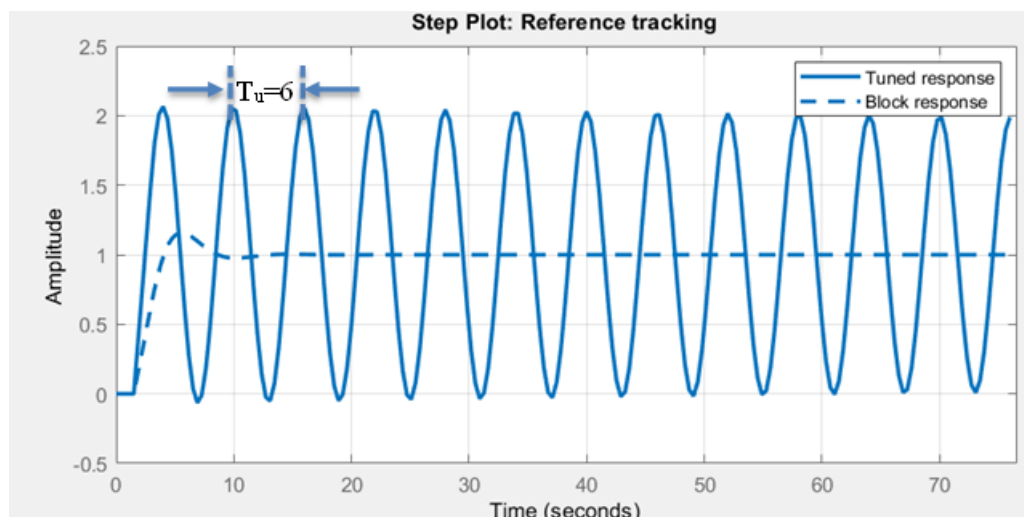


Figure 12 illustrates the determination of the parameter K_u using the Ziegler-Nichols method by bringing the system to the brink of instability.

proportional controller (P) that places the system on the boundary of stability and instability.

According to Figure 12, this value is equal to 2.43 for the modeled train, and the value of T_u is equal to 6.

8. Conclusion

This article introduces an innovative method for simulating train operations in the Shiraz subway system. The approach involves constructing a neural network model based on real data, which takes into account various factors influencing train movement. Additionally, a Fuzzy-PID controller is developed using empirical train operation insights and a fuzzy inference system. The system's effectiveness is verified through tests involving 300 train movements on the Shiraz city train line. This method represents a significant step towards designing a comprehensive Automatic Train Operation (ATO) system that considers all relevant parameters. Its adaptability to unforeseen conditions enhances its utility, and the model can be applied to simulate train behavior in future subway lines with similar trains.

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