



## Parametric Study on Railway Fastening System Response Subjected to Different Axle Load

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### ABSTRACT

There are several factors causing fastening systems to deteriorate faster than the designed life. The high repetitive loads from a moving train being one of the main factors, track irregularities, design and installation defects of track components, non-uniform rail support stiffness, are some of the factors causing fastenings systems to deteriorate faster than the theoretical design life span. The non-uniform support structure induces higher dynamic forces into the track system, leading to early failure of the fastening system. The dynamic forces from the wheel-rail contact transfer to the track subgrade through the fastening system. The fastening acts as a damping to the track system but itself experiences high magnitude undamped dynamic forces. The mechanical properties of the fastening system are the most important characteristics that directly determine the long-term performance of this component. A finite element analysis was carried out to analyze the influence of mechanical properties and geometries of the fastening system on its deterioration. The analysis is done to determine the response of the fastening system under different loading scenarios by changing the mechanical properties. The parameters include: density, young's modulus, yield strength and ultimate strength. A finite element model of a track system built is composed of standard rail (UIC60), rail pad, abrasion plate, rail clip, bolt, insulator and half sleeper, all modeled as solid elements. The model is developed and analyzed using finite element package ANSYS software. The result shows that the mechanical properties and thickness have a great effect on the fastening system components' deterioration. It has been shown that the density has the littlest effect among other parameters and, on the other hand, the young's modulus has seen to have a great effect among the parameters. Finally, the properties of a conceptual rail-fastening system for heavy hauls have been identified. The results presented in this study add more knowledge of mechanistic design theory for fastening systems.

## 1. Introduction

Nowadays there is a high increased demand for heavy hauls and high-speed trains due to the fact that the transportation of goods is shifted to

railway transportation means. Especially in Africa, where countries are seeking to connect most of the ports and big cities by railway lines, so that they can minimize the cost and increase the time taken to the final destination as well as

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the traffic volume. Due to this demand, the railway system components are experiencing unusual deterioration simply because most of them are not designed to withstand these changes. One of the railway systems that are experiencing unexplainable deterioration is the fastening system components. There are several things that are causing the deterioration of fastening systems; apart from the high repetitive forces from a moving railcar, rail seat deterioration, the dynamic forces induced by high-speed railway, the defects of railways, the non-uniform stiffness of track and many others, the mechanical properties of fastening system components may affect the deterioration or reduce the life span of the components. It has been proved that a non-uniform track bed can reduce fatigue life by up to 100 times in comparison with the behavior expected for a uniform track bed. (C. Nkundineza and J.A Turner [1], M.J. Gutierrez et al [2]. This also means that the fastening is among the components affected with that reduction. It has been shown that low stiffness (one of the mechanical behaviors) induces high bending stress (high deformation) of railway components including the fastening system while high stiffness causes high impact forces from the wheel loads (Frohling. R, et al [3], Lundqvist. A and Dahlberg. T [4]. Mechanical properties are the most important characteristics that directly determine the long-term performance of fastening system components, especially the stiffness as it is closely related to the degree of wear fastening system components experience, and the resulting life of the system [2]. As it has been discussed above, having a very low and a very high stiffness of one of the fastening systems, causes the problems. For example, if a very soft elastic fastener (very low stiffness) is used in conjunction with a stiff pad, the rail will not have continuous support during loading cycles, which can cause unwanted impacts and accelerated component wear. Conversely, if a very rigid fastener (very high stiffness) is used in conjunction with a very soft pad, the rail will not be significantly displaced within the rail seat, but the softer pad will wear in an accelerated manner [2]. This is due to the fact that the railway is acting as a system not as an individual component.

The parametric study has been conducted by several researchers; the parameters include: young's modulus, Poisson's ratio Yin Gao [5],

the acceleration of the wheel, elastic modulus of the clips, coefficient of friction between the rail and rail pads, and spacing between sleepers (Zijian Zhang et al [6]), stiffness of the rail fastening, thickness and stiffness of the track slab, the cement emulsified asphalt (CA) mortar cushion, and the concrete supporting layer ,Lu Sun et al [7].A further analysis is needed to account many parameters and analyze their effect on fastening system components not only on individual component like indicated in previous researches. The aim of this research is to perform the parametric study on fastening system components and analyze the response when subjected to different axle loads. And the results from this research will add on more theory and better understanding of mechanical behavior of fastening systems towards mechanist design, which will help in developing a comprehensive design methodology based on detailed structural analysis.

## 2. Methodology

A finite element analysis/simulation was carried out using ANSYS to analyze the influence of mechanical properties on fastening system deterioration. It was also done to determine the response of the fastening system under different loading scenarios by changing the mechanical properties. The parameters include: density, young's modulus, poisson's ratio, yield strength and ultimate strength. A model is composed of standard rail (UIC60), rail pad, abrasion plate, rail clip, bolt, insulator and half sleeper. Since the multi-sleeper model is computational expensive and it is not computationally efficient to use the multi-sleeper model when running parametric analysis, the sub-model of a half-sleeper with fastening system is alternatively chosen in Figure 1. The full sleeper was not considered to minimize the computational time. In order to avoid contact between rail and shoulder, a gap of 5mm was set. The dimensions of the above-mentioned components are described in Table.1.

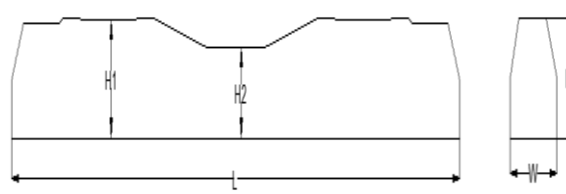


Figure 1. Sub-model of sleeper

The mesh sensitivity analysis was performed as indicated in Fig.3. from the figure it is seen that by increasing the number of elements, there is not much difference in total deformations, while there is a slight difference in equivalent stress, which is very small. To minimize the computation time, the number of elements 104358 was used in this present work.

Table 1. Components dimensions (USA HHS 36/6 RAIL.ONE GmbH 2014)

Parameters	Unit
Concrete grade	C 50/60
Sleeper length (L)	2600 mm
Sleeper width (W)	272 mm
Sleeper height (H)	252 mm
Height of center of rail base ( $h_1$ )	248 mm
Height of sleeper center ( $h_2$ )	190 mm
Rail dimensions	UIC60
Rail pad dimensions	150x160x6 mm
Abrasion plate dimensions	150x170x7 mm
Shoulder dimensions	160x45x26 mm
Rail clip diameter	16 mm
Bolt diameter	24 mm

The boundary conditions are such that the sleeper is fixed at its bottom; the rail is free at its ends and is tied with the sleeper. The details of boundary conditions are indicated in Fig.2. For simplicity, bonded method was applied to the contact interfaces. For the analysis settings, the time step was set to 1 second, the number of steps was 10, the time step was defined by time and for solver, an iterative method was chosen. For all component materials, the elasto-plastic bilinear isotropic hardening constitutive model was used in the analysis to define material properties and for all components their tangent moduli were 10% of young's modulus of each component. The wheel load is applied in the middle of the rail on an assumed surface area of 1 cm<sup>2</sup>. (Using Hertzian theory it is assumed that the contact area between rail and wheel is around 1 cm<sup>2</sup>). The model is shown schematically in Figure.1. Then, the mechanical properties of rail fastening system are changed systematically and are chosen randomly within the realistic range as shown in Table.3 and the influence of mechanical properties on fastening system components is investigated. For the first 3 steps, the clamping force of 10 kN was gradually applied on the rail clip as a pretension force (Chinese standards [8]) on each bolt (with a 58mm diameter) that connected to the clip before applying the vertical and horizontal loads to the ratio of 0.5 (H/V). Then, after the vertical and lateral loads were gradually applied for the remaining steps up to 10. The vertical forces are presented in Table.2 and its point of application can be seen in Figure.2. In order to carry out the fatigue analysis, the S-N curves for fastening system components used were chosen from literature (V.Kazymyrovych [9], B Esmaeillou et al [10], C. S. Woo & W. D. Kim [11], C.X. Ren et al [12], Elouni Chebbi et al [13])

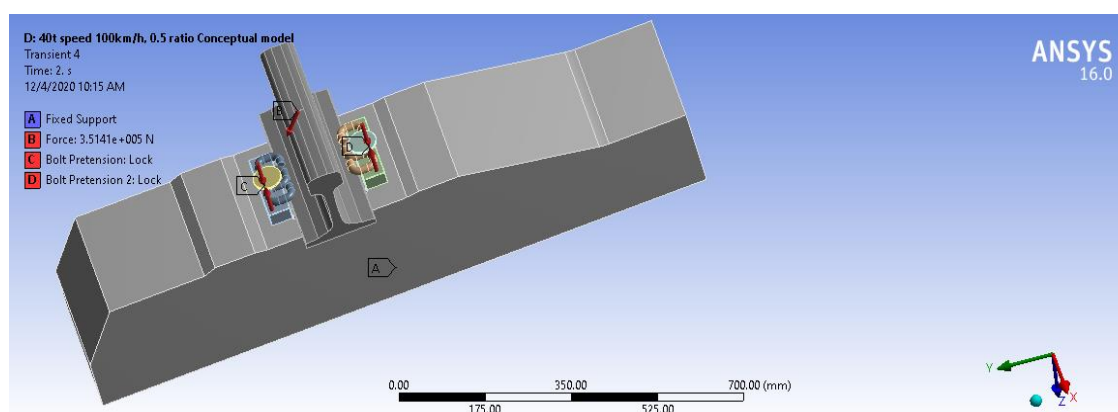


Figure 2. Railway fastening system model with boundary conditions

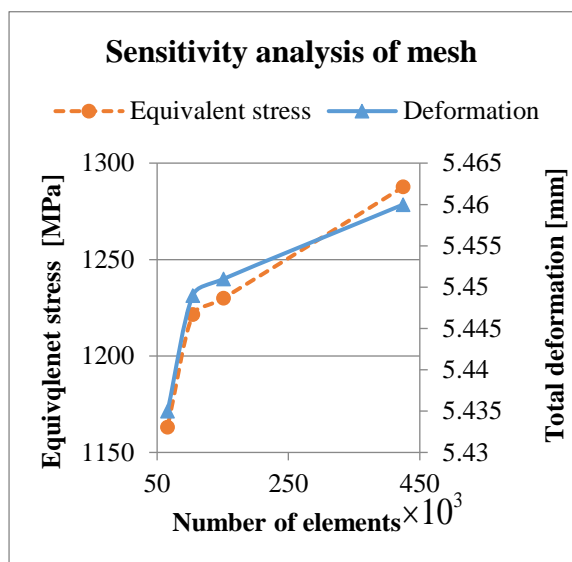


Figure 3. Mesh sensitivity analysis

The analysis of speed increase is really necessary to account for its effect on the fastening system. The speeds of 80, 100, 120 and 160 km/h have been considered. The speeds were chosen because most of the heavy haul

lines are designed not to exceed a speed of 160km/h. The diameter of the wheel was taken as 864mm, the commonly used in heavy haul (Hao et Al 2018). The different wheel load with different axle load used were calculated based on the formula from AREMA to account for the dynamic factor as the speed increases as indicated in equation 1 and 2 and the calculated values are presented in Table.2.

$$\phi = 1 + 5.21 \frac{V}{D} \tag{1}$$

Where V is speed (km/h)

$\phi$  is the dynamic factor

D is wheel diameter (mm) =864mm

$$W = \frac{Axle\ load}{2} \times \phi \tag{2}$$

Where W is the wheel load in kN

In addition to the component dimensions presented in Table 1, material properties are considered as a random variable. The random variables and the different cases considered in this study are shown in Table 3. For all cases, the

Table 2. Vertical load for different speeds

Speed (km/h)	Dynamic factor	Vertical wheel load (kN)			
		Axle load (t)			
		25t	30t	35t	40t
80	1.482	185.25	222.3	259.35	296.4
100	1.603	200.38	240.45	280.52	320.6
120	1.723	215.38	258.45	301.52	344.6
160	1.964	245.5	294.6	343.7	392.8

Table 3. Material properties of the model components for different cases

Cases	Properties	Rail	Rail-pad	Abrasion plate	Rail clip	Bolt	Shoulder	Concrete
Case 1-7	Density (kg/m <sup>3</sup> )	7000-7500	950-1800	950-1800	4500-8500	4500-8500	950-1800	2300-2600
	Young's Modulus (GPa)	150-220	0.055-1	2-5	100-220	100-220	2-4.5	25-50
	Yield Strength (MPa)	800-1600	3-25	40-85	800-1350	400-1300	40-80	Compressive strength (35-55)
	Ultimate Strength (MPa)	850-1800	20-70	50-120	900-1500	600-1500	50-120	
	Poisson's Ratio	0.3	0.49	0.39	0.29	0.3	0.39	0.2

thicknesses of other components are taken as constant. Moreover, the thicknesses of the plate and rail pad are taken as 7mm and 6mm respectively.

### 3. Results and Discussions

#### 3.1. Analysis by varying the parameters of fastening system components

The outputs of 7 cases considered, maximum deformation of fastening system with different material properties at a maximum axle load of 40t and considering the maximum lateral to vertical axle load ratio of 0.5 are shown in Table 4 and the graph for the seven cases with different axle load are presented in Figure 4&5. The results shown in Table 5 are from simulation using ANSYS software for changing the parameters of fastening system components. In this analysis it has revealed that the rail pad is the fastening component that has the highest reduction in deformation. This is because the more increasing in the mechanical properties of rail pad, especially young's modulus, the more it becomes stiffer and reduces its damping and attenuation and induces the reduction in deformation. The other components also undergo considerable changes. Based on results presented in Table 4 and Figure 4, by considering the 25t axle load from case 1 to case 7, the plate reduces to a percentage of 44%, rail clip to 76.9%, bolt to 64.7%, shoulder to 77.7% and on top of the sleeper to 36%. The total deformation is reduced to 54.04 %. In this

analysis, by considering case 7, the results show that when the railway track is subjected to an axle load from 25t to 40t, the overall deformation is increased to 60%. The above observations show that the mechanical properties and axle load have a considerable effect on fastening system deterioration since deformations change based on them. It has been revealed that the young's modulus is the most mechanical property that influences the fastening system deterioration. In addition to that, axle load also has more effects on fastening system deterioration. Hence, during the design it has to be taken into account. During the manufacturing of fastening system components, this has to be taken into account by designing the fastening components based on axle load and mechanical properties of the fastening system components. In order to counter this effect, the component design has to match with the applied loads and consider the mechanical properties.

As shown in Figure 5, the deformations of the rail pad, rail clip, bolt and shoulder are significant. The deformation of the abrasion plate and at the top of the sleeper is almost constant and the variation is insignificant.

Hence, a general empirical equation for the estimation of deformations of the rail pad, rail clip, bolt and shoulder as a function of material properties of different random variables are proposed and given in Equations (3) to (6).

Table 4. Deformation of components

Cases	Maximum deformation (mm)					
	Rail-pad	Abrasion plate	Rail clip	Bolt	Shoulder	Top of sleeper
Case 1	1.44	0.28	0.832	0.478	0.688	0.149
Case 2	0.844	0.24	0.564	0.388	0.459	0.140
Case 3	0.615	0.221	0.427	0.3145	0.342	0.131
Case 4	0.386	0.202	0.29	0.241	0.225	0.122
Case 5	0.371	0.183	0.232	0.204	0.175	0.111
Case 6	0.299	0.184	0.219	0.194	0.165	0.110
Case 7	0.248	0.162	0.191	0.142	0.146	0.096

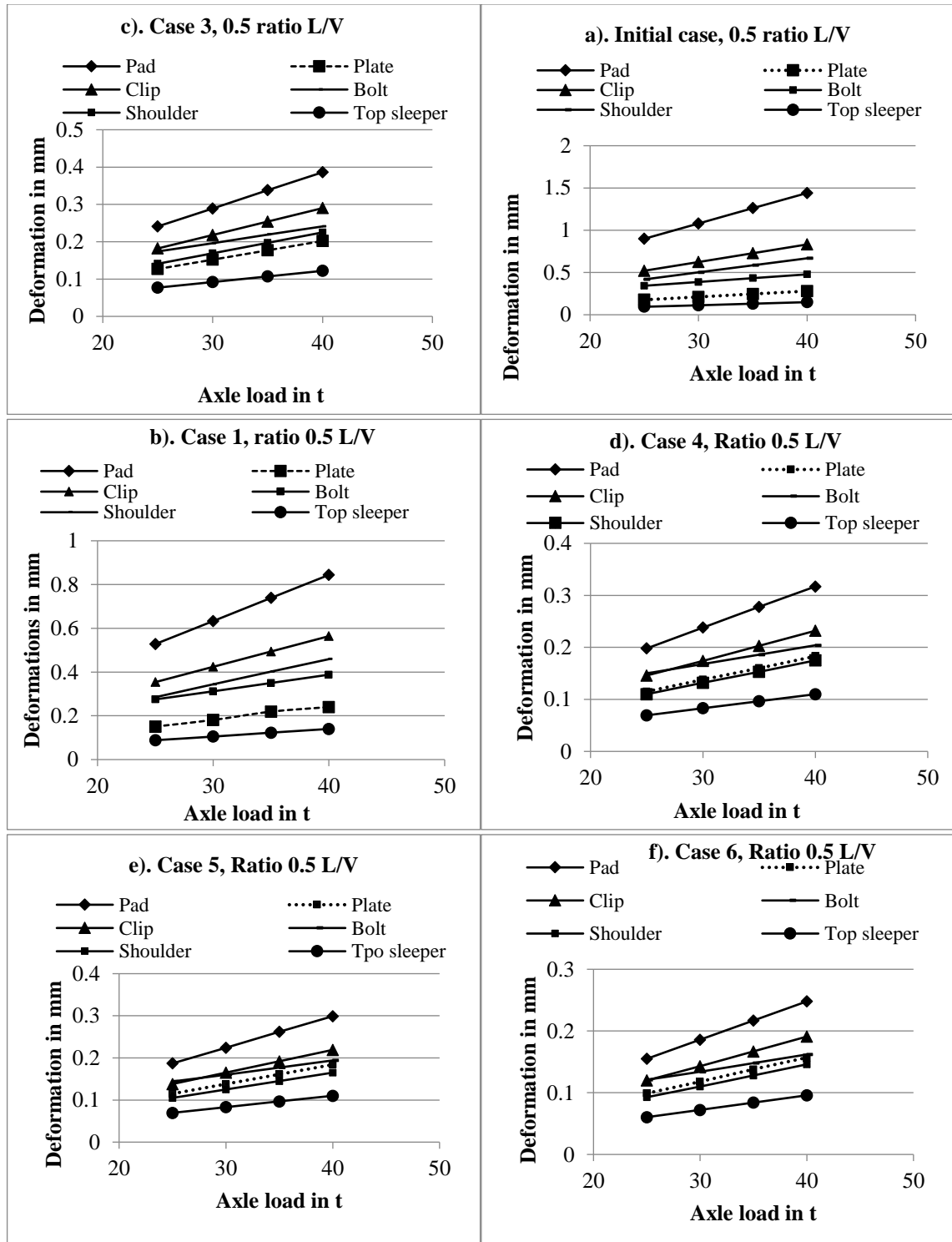


Figure 4. Deformation of fastening system with different material properties subjected to different axle load: a). Initial case, 0.5 ratio L/V, b). Case 1, ratio 0.5 L/V, c). Case 3, 0.5 ratio L/V, d). Case 4, Ratio 0.5 L/V, e). Case 5, Ratio 0.5 L/V and f). Case 6, Ratio 0.5 L/V

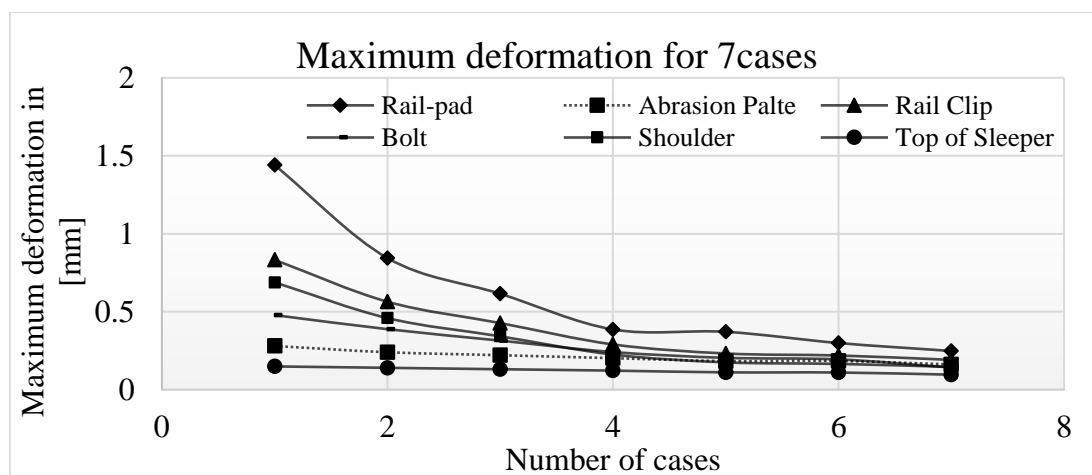


Figure 5. Deformations of fastening system components for 7 cases

$$\Delta_{RP} = (-0.075 \frac{\rho_{RP}}{\rho_S} - 1.56EP - 38.86E_S - 2.35 \frac{FuR}{FyR} + 430.17 \frac{FuRC}{FyRC} + 498.27 \frac{FuB}{FyB} + 3.33 \frac{FuS}{FyS})P * 10^{-4} \quad (3)$$

$$\Delta_{RC} = (-0.019 \frac{\rho_{RP}}{\rho_S} - 0.37EP - 52.09E_S + 0.81 \frac{FuR}{FyR} + 152.27 \frac{FuRC}{FyRC} + 187.27 + 1.98 \frac{FuS}{FyS})P * 10^{-4} \quad (4)$$

$$\Delta_B = (-0.01 \frac{\rho_{RP}}{\rho_S} - 0.15EP - 20.94E_S + 6.22 \frac{FuR}{FyR} + 62.68 \frac{FuRC}{FyRC} + 49.55 \frac{FuB}{FyB} + 0.7 \frac{FuS}{FyS})P * 10^{-4} \quad (5)$$

$$\Delta_S = (-0.014 \frac{\rho_{RP}}{\rho_S} - 0.44EP - 40.47E_S + 4.74 \frac{FuR}{FyR} + 129.38 \frac{FuRC}{FyRC} + 135.56 \frac{FuB}{FyB} + 1.69 \frac{FuS}{FyS})P * 10^{-4} \quad (6)$$

Where:

- P: is the axle load [t]
- $\Delta_{RP}$ : Deformation of rail pad in [mm]
- $\Delta_{RC}$ : Deformation of rail clip in [mm]
- $\Delta_B$ : Deformation of bolt in [mm]
- $\Delta_S$ : Deformation of shoulder in [mm]
- $\rho_{RP}$ : Density of rail pad [kg/m<sup>3</sup>]
- $\rho_S$ : Density of shoulder in [kg/m<sup>3</sup>]
- EP: Young's modulus of rail pad in [GPa]
- ES: Young's modulus of shoulder in [GPa]
- FuR: Ultimate strength of rail pad in [MPa]
- FyR: Yield strength of rail pad in [MPa]
- FuRC: Ultimate strength of rail clip in [MPa]
- FyRC: Yield strength of rail clip in [MPa]
- FuB: Ultimate strength of bolt in [MPa]
- FyB: Yield strength of bolt in [MPa]
- FuS: Ultimate strength of shoulder in [MPa]
- FyS: Yield strength of shoulder in [MPa]

### 3.2. Sensitivity Analysis

Sensitivity analysis is the study of how the variation (uncertainty) in the output of a statistical model can be attributed to different variations in the inputs of the model. It is used to select the order of importance of random variables. The sensitivity of each random variable is represented by the squared value of the partial coefficient of correlation ( $r_p^2$ ). For uncertainty analysis, the sensitivity factor  $\alpha_i$  based on the first-order approximation second-moment method [14] is used. To determine the effects of random variables, the sensitivity factor is obtained as follows.

$$\alpha_i = \frac{\partial F \bar{x}_i}{\partial x_i \bar{F}} \quad (i = 1, 2, 3, \dots, n) \quad (7)$$

- Where,
- $\alpha_i$ : sensitivity factor of random variable  $i$
- $F$ : function with statistical variations
- $\bar{F}$ : mean of  $F$
- $x_i$ : random variable  $i$
- $\bar{x}_i$ : mean of  $x_i$

The sensitivity factor  $\alpha_i$  is a kind of index to estimate the contribution of the uncertainty of  $x_i$  to the uncertainty of  $F$ . The contributions of the uncertainty of each random variable are obtained by multiplying the sensitivity factor by its coefficient of variation [14].

$$U_{x_i, F_i} = \alpha_i (COV_i) \tag{8}$$

Where,

$U_{x_i, F_i}$  : contributions of the uncertainty of random variable  $i$

$COV_i$  : coefficient of variation for random variable  $i$

$\alpha_i$  : sensitivity factor of random variable

The sensitivity and uncertainty factors for the seven random variables, for maximum deformation of the rail-pad are computed and shown in Table 5.

In Table 5, the negative sign implies that the effect of the random parameter is indirectly proportional to the deformation, which reduces the deformation on the rail-pad. Hence, the rail pad is the component of highest reduction of deformation.

### 3.3. Analysis by varying the thickness of rail pad and abrasion plate.

Another analysis was carried out by changing the thickness of the rail pad and abrasion plate by keeping other component dimensions constant.

Table 5. Sensitivity and uncertainty factors for Rail-Pad

Random variables	Sensitivity factor	COV (%)	Uncertainty factor
$\frac{\rho RP}{\rho s}$	-3.9699	9.41	-37.36
$EP$	-2.0554	11.49	-23.62
$E_s$	-0.8699	24.82	-21.584
$\frac{FuR}{FyR}$	-0.066	34.33	-2.2676
$\frac{FuRC}{FyRC}$	2.80651	8.29	23.2667
$\frac{FuB}{FyB}$	4.23514	7.37	31.2089
$\frac{FuS}{FyS}$	0.80216	23.73	19.0341

The analysis was carried out to assess the effect of thickness on the above-said components. The results of 2 cases studied are presented in Figure.6. After the analysis, it has shown that the thickness has a great effect on the deformation not only on the considered components, but also on the overall system. It has also been shown that an increase in axle load has a great effect on the overall performance of railway track. According to the results, the rail pad has the most significant effect on deformation reduction amongst the components. In order to counter this effect, the component design has to match with the applied loads.

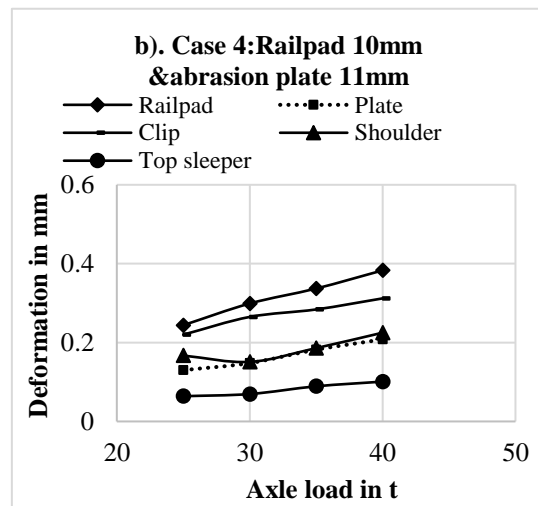
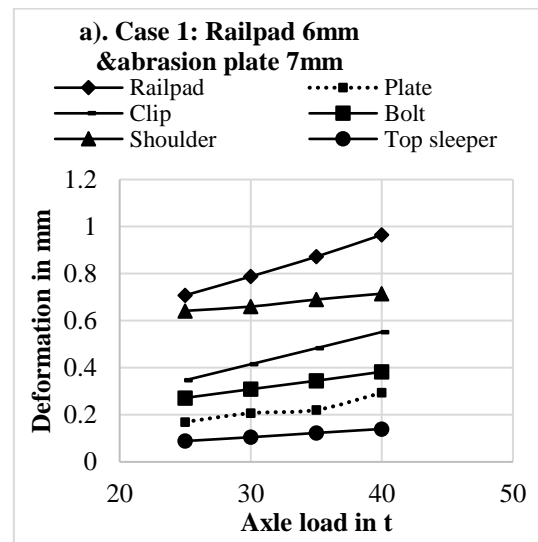


Figure 6. Deformation of rail pad and abrasion plate by changing dimensions. Case 1: Rail pad 6mm & abrasion plate 7mm. b). Case 2: Rail pad 10mm & abrasion plate 11mm



### 3.4. Fatigue Analysis results

After carrying out the parametric and geometric study on rail fastening components, the fatigue analysis was carried out in order to present the conceptual model. The conceptual model properties are presented in Table 6. They were chosen after several simulations of parametric studies. The fatigue analysis indicates how the chosen conceptual model will resist on repeated loading. The results are presented in Figures 7,8&9.

The fatigue analysis has been performed considering the design life of  $10^9$  cycles. From this analysis, the life analysis indicated that rail can resist up to 964 cycles minimum, rail clip up to  $9 \times 10^8$  cycles, railpad up to 87 cycles, abrasion plate up to  $4 \times 10^5$  cycles, sleeper up to  $10^9$  cycles except at the top that fails before reaching the design life, bolts up to  $4 \times 10^5$  cycles and insulator up to  $2 \times 10^7$ . The safety factor analysis indicates that rail (at the point of load application and rail web) will fail before reaching the design life, rail clip will not fail,

railpad will fail around the edges, abrasion plate will not fail, sleepers only at the top (rail seat) will fail, bolt will fail at the top (small surface) and the insulator will likely fail at the point where rail clips seats. The sensitivity analysis indicated that for rail when the load is beyond 15% of current load, the life turns to 0, for rail clip the life cycle turns to 0 when, it is 280% of the current load, for the railpad, the life turns to 0 at 95% of current load, for abrasion plate the life cycle turns to 0 when the load is 15% of current load, for sleeper to 20% of current load, for bolt to 60% and for the insulator to 110% of current load. The above observations indicate that by considering the properties in Table 6, the life span of the fastening system as well as the overall performance of tract structure increase considerably.

Table 6. Properties of selected for conceptual model

Material	Density (kg/m <sup>3</sup> )	Young's Modulus (MPa)	Poisson's ratio	Yield strength (MPa)	Ultimate strength (MPa)	Dimensions (mm)
Rail	8000	210	0.3	1300	1350	Standard UIC60
Railpad	1100	0.66	0.49	14	45	160x150x8
Abrasion plate	1300	4	0.39	78	95	170x160x9
Rail clip	7800	180	0.29	1150	1250	16mm of dia.
Bolt	7800	210	0.3	850	1050	24mm of dia.
Shoulder	1300	4	0.39	78	95	160x45x26
Concrete	2500	40	0.2	-	-	C50/60

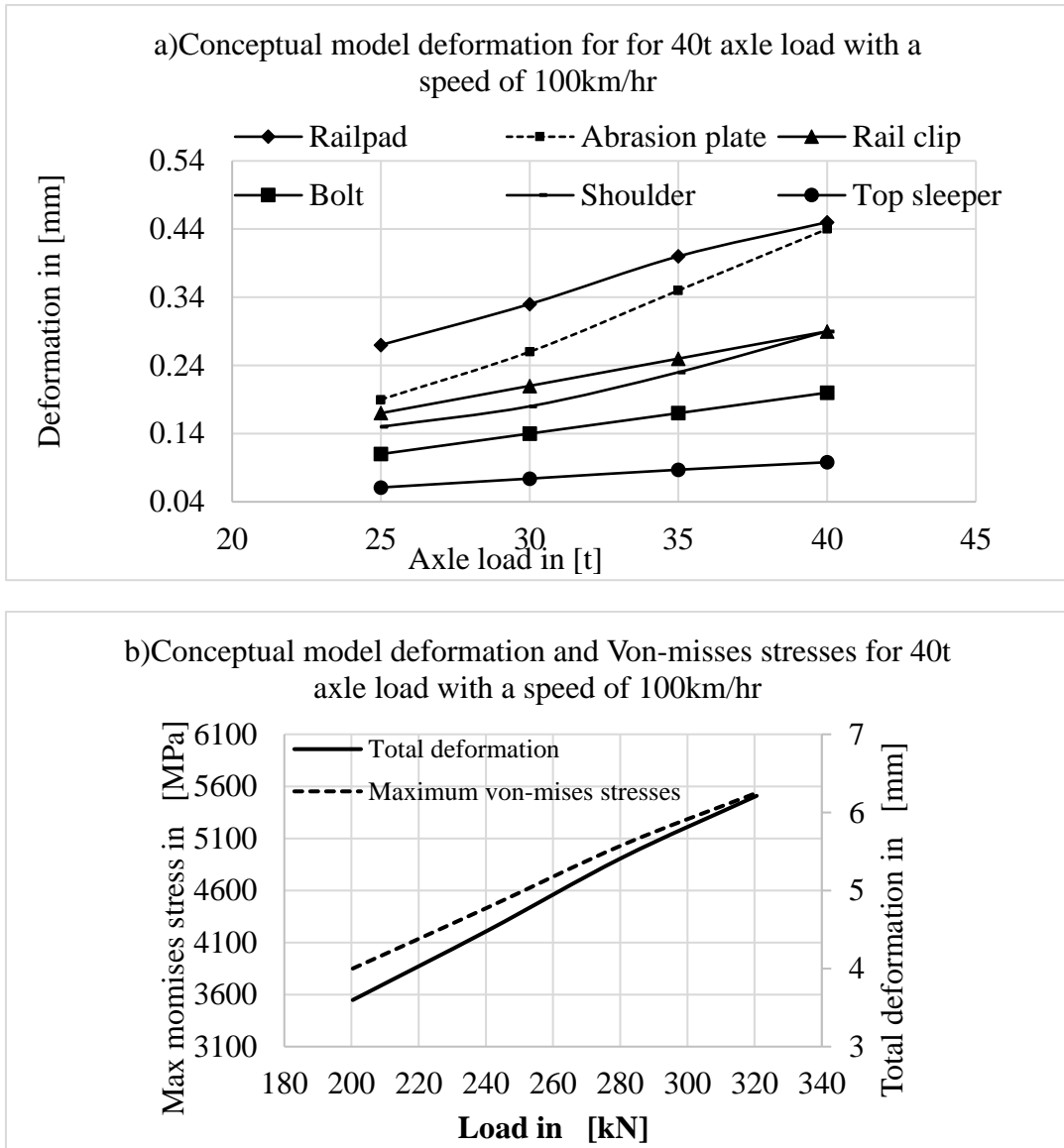


Figure 7. Conceptual mode analysis results a) Conceptual model deformation for 40t axle load with a speed of 100km/hr., b) Conceptual model deformation and Von-mises stresses for 40t axle load with a speed of 100km/hr.

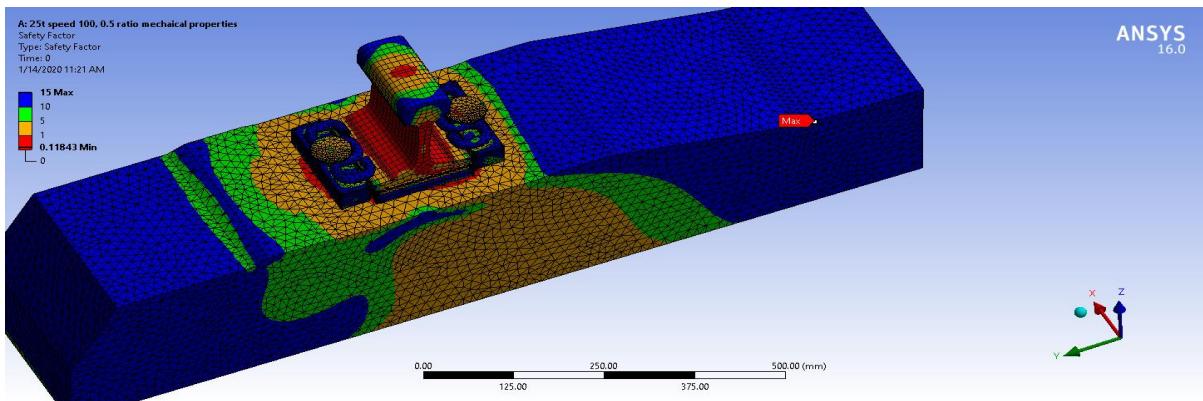


Figure 8. Safety factor for the conceptual model for 25t axle load with a speed of 100 km/h considering to the ratio of 0.5 h/v load

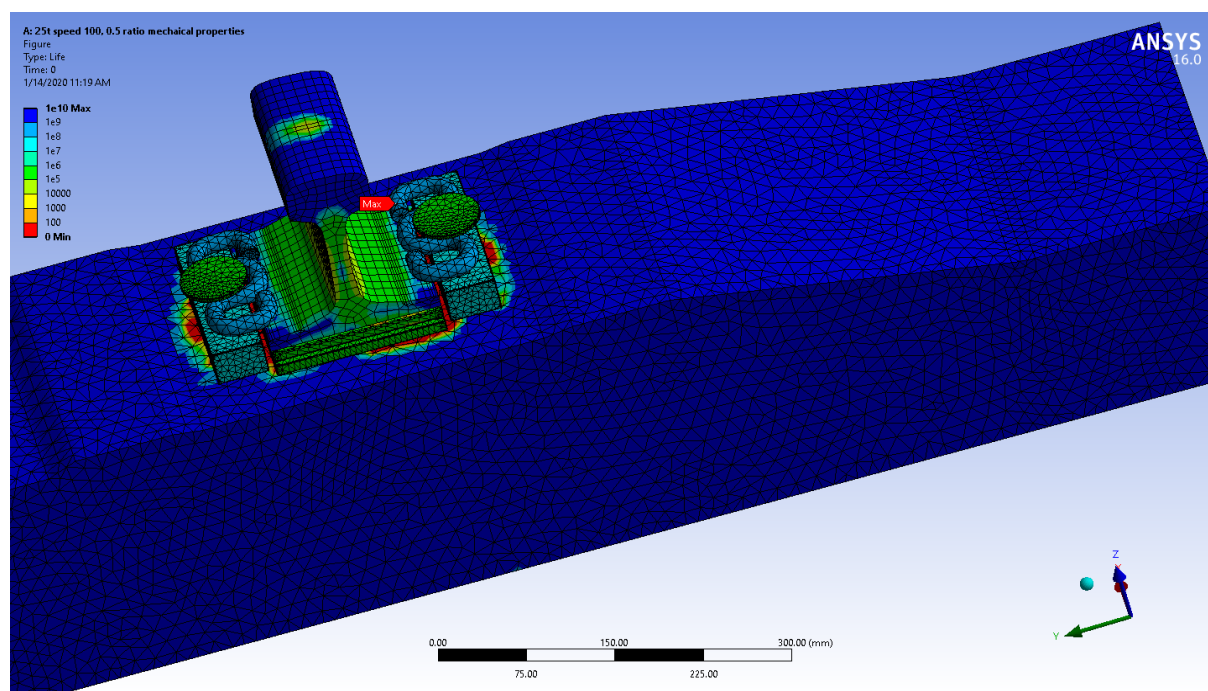


Figure 9. Life for the conceptual model for 25t axle load with a speed of 100 km/h considering the ratio of 0.5 h/v load

### 3.5. Modal validation

To validate the results, the comparison of the model results and test results from literature was done. The comparison was compared with the work done by Z.Chen et al [15]. The loading conditions were the same as the ones used in literature. A 10kN lateral load was gradually applied to the rail head surface, as in the literature. The results of comparison are presented in Fig.10. As is indicated, there is a good agreement between the model and test results.

### 4. Conclusions

Parametric study using numerical analysis have been conducted for heavy hauls with different axle loads, and the following conclusions have been drawn:

1. It has shown that the mechanical properties and thickness have a great effect on the fastening system components deterioration.
2. It has been revealed that the rail-pad has the highest reduction of deformation among the other fastening components.
3. The young's modulus has been seen to have a great effect among other parameters. From

fatigue analysis, it has revealed that most of the fastening components can resist up to  $4 \times 10^5$  cycles before failure. And it has also revealed that by increasing the considered loads by 25%, life decreases to 0.

4. In this study the properties of a rabostic model of the rail-fastening system for heavy hauls have been identified and it has been revealed that the vulnerable components are rail (at the point of application and at the top of the sleeper). Other may resist up to  $4 \times 10^5$  cycles. The properties of the selected model are seen in table 6.

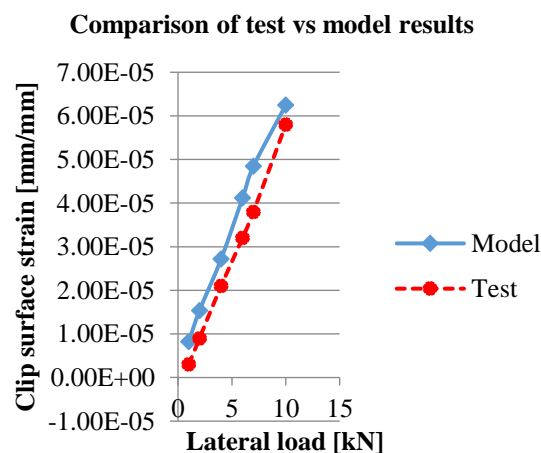


Figure 10. Validation model

It is recommended that the manufacturers design the materials based on mechanistic design not on evaluative tests. The author recommends also carrying out the research on load distribution and mathematical models for different components of fastening systems.

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