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Integrated timetable rescheduling and rolling stock reassignment for an urban rail transit system during large-scale disruptions

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Railway transport systems are critical for the economy's competitiveness and the mobility of people and goods. These systems are increasingly important in urban public transportation networks because of their high capacity, punctuality, and low energy consumption. In actual operations, unavoidable disruption often delays the original train timetable. When trains are delayed, dispatchers adapt the impacted train timetables to the perturbations. This paper captures real-time traffic management using a mixed-integer linear programming (MILP) model. Flexible stopping is innovatively integrated with delaying and canceling to reschedule a timetable during railway disruptions. The model presented in this paper also considers the assignment of rolling stock (RS) to courses. A real-world instance of London's new Elizabeth line, UK, was used to test the proposed model based on several disruption scenarios. The Elizabeth Line (formally known as Crossrail) will stretch more than 96 kilometers from Reading and Heathrow in the west through central tunnels across to Shenfield and Abbey Wood in the east. The RAS2022 competition provides these problem instances. The model was solved with the Gurobi solver. The computational results from the Gurobi solver show that the performance of the proposed model is appropriate for solving the problem. The model can solve all three evaluation problem instances with good quality within the time limit. Our experimental results found that delay is preferable to canceling a course in our model.

1. Introduction

A major feature of rail transit systems is their high capacity, punctuality, and low energy consumption, which make them a valuable component of urban public transportation. As rail equipment ages, incidents and disruptions occur frequently, often delaying original train timetables [1]. It is necessary to make train dispatching decisions in these circumstances, such as canceling, re-timing, re-platforming, and skipping stops [2]. In a railway system, the route and departure and arrival times of each train are determined by the planning timetable. As a result, dispatchers do not have to adapt the timetable if the trains run according to schedule. However, inevitable disruptions are caused by external or internal incidents such as extended run times, late departures from the depot, or extended dwell times at a station. In a disrupted situation, dispatchers must update the original timetable into an amendment timetable based on the latest information. Therefore, from then on, trains will have to run following the amended timetable instead of the originally planned one.

This paper focuses on this problem for an urban rail transit system on a long and busy railway line based on the problem defined in the RAS2022 competition [3]. To this end, a new mixed integer programming model is formulated to deal with the problem of timetable

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rescheduling. The goal is to adjust trains back to their original schedule as soon as possible by re-timing, skipping stops, and canceling trains. The objective is to minimize the total penalties for skipped stops, destination delays, and passage frequency.

Traffic management to reduce the impact of disruptions has been attracting much attention in recent years. It is expected to study timetable rescheduling and rolling stock reassignment to optimize the operation strategies further simultaneously. Some studies integrated these two problems in the literature to obtain better performances. These papers are reviewed in the following paragraphs:

In 2013, Cadarso et al. [4] formulated a two-step procedure that decided on the timetable and the rolling stock plan using an integrated optimization model accounting for passenger demand behavior. Moreover, their study shows that the rolling stock composition changes according to passengers' behavior. In 2017, Veelenturf et al. [5] proposed a heuristic approach for network-level timetabling and rolling stock rescheduling with dynamic passenger demand. Their focus lies on railway networks in which passengers have the free choice of taking trains; that is, a seat reservation system does not constrain them. Dollevoet et al. [6] proposed an iterative framework in which all three resources, the timetable, rolling stock, and crew, are considered.

In 2021, Wang et al. [1] presented a multi-objective mixed integer linear programming (MILP) model to optimize timetable rescheduling and rolling stock circulation planning. Objectives considered include minimizing deviations from the original timetable, cancellations, and the headway frequency of train services. To enhance computational efficiency, they designed a two-stage algorithm inspired by the experimental disruption management of metro lines. In the first stage, the MILP is solved by only considering key stations to decrease the solution space. The obtained variables in the first stage are then given as input for the optimization problem in the second stage. Finally, they performed their computational experiments on real-world data from the Beijing Subway Line. Long et al. [7] proposed a mixed-integer nonlinear programming model to integrate train timetables and rolling stock plans to manage a large unexpected passenger flow. They used approximate and exact reformulation approaches to transform the nonlinear model into a mixed-integer linear programming model. In 2022, Pascariu et al. [8] proposed an improved train routing selection model, in which the effects of train delay propagation and rolling stock reutilization constraints are considered. A parallel ant colony optimization (ACO) algorithm is also presented to accelerate search space exploration.

The rest of the paper is structured as follows: Section 2 defines the problem statement and its assumptions. Section 3 presents a new mixed-integer programming model for designing real-time traffic management (DRTM). Section 4 presents the computational results obtained from solving the instances. Finally, section 5 describes the conclusions and suggestions for future research.

2. Problem statement

This section explains the problem and its assumptions according to the problem description document of RAS2022 (RAS_PSC_problem_description_v24.pdf) [3]. The assumptions and concepts intended to build the model are as follows:

- A course is a train scheduled in the timetable that shows the nodes the train passes through and the arrival and departure times.
- A train set is referred to as "rolling stock." Excluding split-and-join operations, all considered services and empty moves to/from depots employ the same type of rolling stock (345 class EMUs).
- A duty is a set of courses run by one rolling stock.
- The entire network considered is double-tracked. The number of tracks at certain stations may, however, exceed two. For each station, a set of tracks is considered in each direction.
- A minimum separation time is considered between when a course leaves a track at a station and when the next course enters that track. This assumption ensures that each track is occupied by at most one train at any given time.

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- A minimum run time is considered between when a course enters a station and when it leaves the next station. This time depends on the course's direction and activity at two stations, i.e., pass/pass, pass/stop, stop/pass, stop/stop.

- A minimum time is considered between the departure of a course from one station and its arrival at the next station. This time depends on the distance between the two stations and is calculated from the planning timetable.

- If two courses move from station s_1 to station s_2 in two consecutive sequences, a minimum headway time is considered between the two departure times of courses from station s_1. This time depends on the activity of two courses at two stations.

- We consider a minimum change-end time when one course ends and another begins.

- When one duty set ends and a new one begins for the same rolling stock, a minimum time is considered between them.

- The number of rolling stock in the network is limited.

- The rolling stocks should start from a depot station and return to a depot station at the end. Also, the number of rolling stocks leaving a station must equal the number of rolling stocks entering that station (rolling stock balance).

- A duty must be completed with one set of rolling stock.

- Each course must either stop or pass through its route station. A minimum dwell time according to the planning timetable is considered while stopping at the station. If there is no stop, the time spent entering and leaving the station should be equal.

- A track must be assigned to each sequence of a course.

- Timetable amendments are modifications to the original timetable. Amendments include the following types:

1) Course retiming in a station: modified arrival/departure (stop) times;
2) Replatforming of courses in a station: considering the set of available tracks for that course in that station, the planned station track is modified;
3) Skipped stop at a station: The train skips a stop at a station instead of stopping as planned;
4) Partial cancellations in a station: An intermediate location is "cut" in a partial cancellation; and
5) Total cancellations: An entire course is canceled.

- The penalty for actual traffic divergence from the planned timetable is calculated using the following KPIs:

1) Skipped stops;
2) Destination delays; and
3) The headway is considered in the central part of the network, i.e., between Whitechapel and Paddington.

- The following four types of incidents are considered:

1) Extended run times on an edge;
2) Late departure from a depot;
3) A longer dwell time at a station for all trains; and
4) A longer dwell time at a station for a train.

- The important point is that before taking a timetable snapshot, the courses have taken actual values for their sequences' arrival/departure times in the problem instances. Since the time of the completed course sequences cannot be changed, these sequences have not been considered in our model constraints.

3. The proposed mixed-integer programming model for DRTM

In this section, a new mixed-integer programming model, MDRTM (Model for Designing Real-time Traffic Management), is proposed for rescheduling trains in the event of disruptions. In order to explain the mathematical formulation, it is first necessary to introduce the notations used. The operational constraints and objective functions will be formulated in the next step. The sets and parameters used in the model are illustrated in Table 1 and Table 2, respectively.

All the decision variables used to formulate the problem in the study are listed in Table 3.

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Table 1: Sets used in the MDRTM

Name	Definition	Index
C	Set of courses in the planned timetable (OO/EE)	c, c'
C_{OO}	A subset of *C*; set of *OO* (passenger service) courses in the planned timetable	c, c'
C_{dif}^{sep}	The set of pairs (*c*,*c'*) with a maximum 5-minute absolute difference in the planned departure time of *c* and the planned arrival time of *c'*	c, c'
C_{dif}^{head}	The set of pairs (*c*,*c'*) with a maximum 5-minute absolute difference in the planned departure time of *c* and *c'*	c, c'
CE	Set of virtual courses to complete the sequence	c, c'
CS	Set of virtual courses to start the sequence	c, c'
D	Set of duty in the rolling stock duty	d, d'
R	Set of rolling stock	r
TC_c	Set of sequences related to the course *c* in the planned timetable	t, t'
TR_r	Set of sequences related to the rolling stock *r*	q
TD_d	Set of sequences related to the duty *d* in the planned timetable	t, t'
S_c	Set of stations related to the course *c* in the planned timetable	s, s'
S_d	Set of depot stations in the network	s, s'
Tr_s^{WB}	Set of tracks related to the station *s* and direction *WB*	k
Tr_s^{EB}	Set of tracks related to the station *s* and direction *EB*	k
Seq_{EB}	Set of pairs *(c,t)* arriving at station *WCHAPXR* in direction *EB*	c, c', t, t'
Seq_{WB}	Set of pairs *(c,t)* arriving at station PADTLL in direction WB	c, c', t, t'

Table 2: Parameters used in the MDRTM

Name	Definition
M	A large enough positive number
id_s	The ID of the station *s*
bsv_s^{ct}	Base station value for sequence *t* of course *c* at station *s*
ssf_s^{ct}	Station stop factor for sequence *t* of course *c* at station *s*
st^{ct}	Station associated with sequence *t* of course *c*.
dir_c	The direction of course *c* in the planned timetable
$ht_{c't'}^{ct}$	The threshold headway for sequence t of course c and sequence t' of course c'
num_RS	Number of active rolling stocks
ca_{dt}^{c}	If course *c* is assigned to sequence *t* of duty *d*, it is equal to 1; otherwise, 0.
ss_c	Start station of course *c* in the planned timetable
se_c	End station of course *c* in the planned timetable
y_s^{ct}	If station *s* corresponds to sequence *t* of course *c*, it is equal to 1; otherwise, 0.
l_dwell^{ct}	Pre-determined dwell time in station st^{ct} *in* the planned timetable
l_Edwell	Pre-determined extended dwell time
l_run^{ct}	Pre-determined running time from station st^{ct} to station st^{ct+1} *in* the planned timetable
l_Erun	Pre-determined extended running time
l_sep^{ct}	Pre-determined separation time in the station st^{ct}

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Table 2: Parameters used in the MDRTM

Name	Definition
c_inc_c	If the end time of course c is before the start time of the incident, it is equal to 0; otherwise, 1. In other words, if all the sequences of course c have occurred before the start time of the incident in the realized_schedule table, this parameter takes a zero value.
$retd^{ct}$	Actual arrival time of sequence t of course c *in* the realized_schedule table. This parameter is equal to -1 for sequences that have not happened yet.
$reta^{ct}$	Actual departure time of sequence t of course c *in* the realized_schedule table. This parameter is equal to -1 for sequences that have not happened yet.
ota^{ct}	Original arrival time of sequence t of course c in the planned timetable
otd^{ct}	Original departure time of sequence t of course c in the planned timetable
act^{ct}	Activity of sequence t of course c
$l\_runtime\_1skipi^{ct}$	Minimum run time for sequence t of course c if the planned stop at station st^{ct} is skipped.
$l\_runtime\_1skipj^{ct}$	Minimum run time for sequence t of course c if the planned stop at station st^{ct+1} is skipped.
$l\_runtime\_2skip^{ct}$	Minimum run time for sequence t of course c if the planned stop at two stations st^{ct} and st^{ct+1} are skipped.
$l\_runtime\_Nskip^{ct}$	Minimum run time for sequence t of course c, if stopping as planned at two stations st^{ct} and st^{ct+1}.
$l\_Nheadway^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if stopping as planned at four stations st^{ct} ,st^{ct+1},$st^{c't'}$, and $st^{c't'+1}$.
$l\_4headway^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at four stations st^{ct} ,st^{ct+1},$st^{c't'}$, and $st^{c't'+1}$ are skipped.
$l\_1headway\_cs^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at station st^{ct} is skipped.
$l\_1headway\_csp^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at station st^{ct+1} is skipped.
$l\_1headway\_cps^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at station $st^{c't'}$ is skipped.
$l\_1headway\_cpsp^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at station $st^{c't'+1}$ is skipped.
$l\_2headway\_cscsp^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at two stations st^{ct} and st^{ct+1} are skipped.
$l\_2headway\_cscps^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at two stations st^{ct} and $st^{c't'}$ are skipped.
$l\_2headway\_cscpsp^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at two stations st^{ct} and $st^{c't'+1}$ are skipped.
$l\_2headway\_cspcps^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at two stations st^{ct+1} and $st^{c't'}$ are skipped.
$l\_2headway\_cspcpsp^{ct}_{c't'}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at two stations st^{ct+1} and $st^{c't'+1}$ are skipped.

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Table 2: Parameters used in the MDRTM

Name	Definition
$l\_2headway\_cpscpsp_{c't'}^{ct}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at two stations $st^{c't'}$ and $st^{c't'+1}$ are skipped.
$l\_3headway\_cs_{c't'}^{ct}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at three stations st^{ct+1}, $st^{c't'}$, and $st^{c't'+1}$ are skipped.
$l\_3headway\_csp_{c't'}^{ct}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at three stations st^{ct}, $st^{c't'}$, and $st^{c't'+1}$ are skipped.
$l\_3headway\_cps_{c't'}^{ct}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at three stations st^{ct}, st^{ct+1}, and $st^{c't'+1}$ are skipped.
$l\_3headway\_cpsp_{c't'}^{ct}$	The minimum headway time between sequence t of course c and sequence t' of course c', if the planned stop at three stations st^{ct}, st^{ct+1}, and $st^{c't'}$ are skipped.
st_start_Erun	The start station associated with the extended run time incident
st_end_Erun	The end station associated with the extended run time incident
t_start_Erun	The start time of the extended run time incident
t_end_Erun	The end time of the extended run time incident
t_start_Edwell	The start time of the extended dwell time incident
t_end_Edwell	The end time of the extended dwell time incident
st_Edwell	The station associated with the extended dwell time incident

Table 3: Decision variables used in the MDRTM

Name	**Definition**	**Domain**
RTA^{ct}	The rescheduled arrival time of sequence *t* corresponding to course *c*	$\mathbb{R}_{+}$
RTD^{ct}	The rescheduled departure time of sequence *t* corresponding to course *c*	$\mathbb{R}_{+}$
$Y\_PF_{c't'}^{ct}$	If the sequence t of course c at the reference station is exactly before the sequence t' of course c', it is equal to 1; otherwise, 0.	$\{0,1\}$
DD_c	The arrival delay time of course *c* at the last station of its actual journey.	$\mathbb{R}_{+}$
$Delay_c$	If the arrival delay time of course *c* at the last station of its actual journey is more than 3 minutes, it is equal to 1; otherwise, 0.	$\{0,1\}$
D^{ct}	The arrival delay time of sequence t corresponding to course c	$\mathbb{R}_{+}$
Pen_D_c	Violation penalty value; If $Delay_c = 1$, it is equal to the amount of arrival delay time of course c at the last station of its actual journey; otherwise, 0.	$\mathbb{R}_{+}$
$Pen\_PF_{c't'}^{ct}$	Violation penalty value; ; If $Y\_PF_{c't'}^{ct} = 1$, it is equal to the amount of the headway penalty for sequence t of course c and sequence t' of course c' at the reference station; otherwise, 0.	$\mathbb{R}_{+}$
RSA_{rqd}	If duty *d* is assigned to sequence *q* of rolling stock *r*, it is equal to 1; otherwise, 0.	$\{0,1\}$
Z_r	If rolling stock *r* is used, it is equal to 1; otherwise, 0.	$\{0,1\}$
ES_{cs}	If station *s* is the last uncancelled station of course *c*, it is equal to 1; otherwise, 0.	$\{0,1\}$

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Table 3: Decision variables used in the MDRTM

Name	Definition	Domain
STC_{dt}	The rescheduled start time of sequence *t* of duty *d*	$\mathbb{R}_{+}$
ETC_{dt}	The rescheduled end time of sequence *t* of duty *d*	$\mathbb{R}_{+}$
$ESRS_{rd}$	If duty *d* is the last duty assigned to rolling stock *r*, it is equal to 1; otherwise, 0.	$\{0,1\}$
ESR_{rd}^{cs}	If station *s* is the last station of course *c* of its actual journey, course *c* is the last course assigned to duty *d*, and duty *d* is the last duty assigned to rolling stock *r*, it is equal to 1; otherwise, 0.	$\{0,1\}$
SSR_{rd}^{cs}	If station *s* is the first station of course *c*, course *c* is the first course assigned to duty *d*, and duty *d* is the first duty assigned to rolling stock *r*, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Dwell^{ct}$	If course *c* stops at station st^{ct} in sequence *t* of its journey, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Pass^{ct}$	If course *c* passes station st^{ct} in sequence *t* of its journey, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Track_{k}^{ct}$	If track *k* is assigned to sequence *t* of course *c*, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Pr_{c}^{c't'}$	If sequence t' of course c' occurs before c , it is equal to 1; otherwise, 0.	$\{0,1\}$
C^{ct}	If sequence *t* of course *c* is canceled, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Skip_{s}^{ct}$	If sequence *t* of course *c* passes through *s* instead of stopping as in the planned timetable, it is equal to 1; otherwise, 0.	$\{0,1\}$
ESC^{ct}	If sequence *t* is the last uncancelled sequence of course *c*, it is equal to 1; otherwise, 0.	$\{0,1\}$
$EDrun^{ct}$	Auxiliary variable; If the sequence *t* of course *c* is after the start time of the extended run time incident, it is equal to 1; otherwise, 0.	$\{0,1\}$
$EArun^{ct}$	Auxiliary variable; If the sequence *t* of course *c* is before the end time of the extended run time incident, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Extended\_run^{ct}$	Auxiliary variable; If the sequence t of the course c is in the time band of the extended run time incident, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Edwell^{ct}$	Auxiliary variable; If the sequence *t* of course *c* is after the start time of the extended dwell time incident, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Edwellp^{ct}$	Auxiliary variable; If the sequence *t* of course *c* is before the end time of the extended dwell time incident, it is equal to 1; otherwise, 0.	$\{0,1\}$
$Extended\_dwell^{ct}$	Auxiliary variable; If the sequence *t* of the course *c* is in the time band of the extended dwell time incident, it is equal to 1; otherwise, 0.	$\{0,1\}$

The formulation of the MDRTM model is as follows:

$$Min\ Z = \sum_{c\in C_{oo}} \sum_{t\in TC_c} \sum_{s|y_s^{ct}=1} bsv_s^{ct} \times ssf_s^{ct} \times Skip_s^{ct} \quad (1)$$
$$+\sum_{c\in C_{oo}} 125 \times Pen\_D_c$$
$$+\sum_{c\in C} \sum_{\substack{t\in TC_c \\ (c,t)\in Seq_{EB}}} \sum_{c'\in C} \sum_{\substack{t'\in TC_{c'} \\ (c',t')\in Seq_{EB}}} 150 \times Pen\_PF_{c't'}^{ct}$$
$$+\sum_{c\in C} \sum_{\substack{t\in TC_c \\ (c,t)\in Seq_{WB}}} \sum_{c'\in C} \sum_{\substack{t'\in TC_{c'} \\ (c',t')\in Seq_{WB}}} 150 \times Pen\_PF_{c't'}^{ct}$$

Subject to:

$$DD_c \le 180 + M(Delay_c) \quad \forall c \in C_{OO} \quad (2)$$

$$Pen\_D_c \le M(Delay_c) \quad \forall c \in C_{OO} \quad (3)$$

$$Pen\_D_c \le DD_c \quad \forall c \in C_{OO} \quad (4)$$

$$Pen\_D_c \ge DD_c - M(1 - Delay_c) \quad \forall c \in C_{OO} \quad (5)$$

$$RTA^{ct} \le RTA^{c't'} + M(1 - Y\_PF_{c't'}^{ct}) \quad (6)$$

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$$\forall c,c' \in C, c \neq c', t \in TC_c, t' \in TC_{c'}, (c,t),(c',t') \in Seq_{EB}$$

$$RTA^{ct} \leq RTA^{c't'} + M(1 - Y\_PF_{c't'}^{ct}) \quad (7)$$
$$\forall c,c' \in C, c \neq c', t \in TC_c, t' \in TC_{c'}, (c,t),(c',t') \in Seq_{WB}$$

$$\sum_{c' \in C \bigcup CE} \sum_{\substack{t' \in TC_{c'} \\ (c',t') \in Seq_{WB}}} Y\_PF_{c't'}^{ct} = 1 \quad (8)$$
$$\forall c \in C \bigcup CS, t \in TC_c, (c,t) \in Seq_{WB}$$

$$\sum_{c' \in C \bigcup CE} \sum_{\substack{t' \in TC_{c'} \\ (c',t') \in Seq_{EB}}} Y\_PF_{c't'}^{ct} = 1 \quad (9)$$
$$\forall c \in C \bigcup CS, t \in TC_c, (c,t) \in Seq_{EB}$$

$$\sum_{c \in C \bigcup CS} \sum_{\substack{t \in TC_c \\ (c,t) \in Seq_{WB}}} Y\_PF_{c't'}^{ct} = 1 \quad (10)$$
$$\forall c' \in C \bigcup CE, t' \in TC_c, (c',t') \in Seq_{WB}$$

$$\sum_{c \in C \bigcup CS} \sum_{\substack{t \in TC_c \\ (c,t) \in Seq_{EB}}} Y\_PF_{c't'}^{ct} = 1 \quad (11)$$
$$\forall c' \in C \bigcup CE, t' \in TC_c, (c',t') \in Seq_{EB}$$

$$RTA^{c't'} - RTA^{ct} \leq ht_{c't'}^{ct} + M(1 - Y\_PF_{c't'}^{ct}) + Pen\_PF_{c't'}^{ct} \quad (12)$$
$$\forall c,c' \in C, c \neq c', t \in TC_c, t' \in TC_{c'}, (c,t),(c',t') \in Seq_{EB}$$

$$RTA^{c't'} - RTA^{ct} \leq ht_{c't'}^{ct} + M(1 - Y\_PF_{c't'}^{ct}) + Pen\_PF_{c't'}^{ct} \quad (13)$$
$$\forall c,c' \in C, c \neq c', t \in TC_c, t' \in TC_{c'}, (c,t),(c',t') \in Seq_{WB}$$

$$\sum_{r \in R} \sum_{q \in TR_r} RSA_{rqd} = 1 \qquad \forall d \in D \quad (14)$$

$$\sum_{d \in D} RSA_{rqd} \leq 1 \qquad \forall r \in R, q \in TR_r \quad (15)$$

$$M(Z_r) \geq \sum_{d \in D} \sum_{q \in TR_r} RSA_{rqd} \qquad \forall r \in R \quad (16)$$

$$\sum_{r \in R} Z_r \leq num\_RS \quad (17)$$

$$RSA_{rqd} + RSA_{rq+1d'} \leq 1 - M(1 - ES_{cs}) \quad (18)$$
$$\forall r \in R, q \in TR_r, d \neq d' \in D, c \neq c' \in C$$
$$s \in S_c, ca_{d|T_d|}^{c} = 1, ca_{d'1}^{c'} = 1, ss_{c'} \neq id_s$$

$$\sum_{q'=q+1}^{|TR_r|} \sum_{d \in D} RSA_{rq'd} \leq M \sum_{d \in D} RSA_{rqd} \quad \forall r \in R, q \in TR_r \quad (19)$$

$$STC_{d'1} - ETC_{d|TD_d|} \geq 420 - M(2 - RSA_{rqd} - RSA_{rq+1d}) \quad (20)$$
$$\forall d \neq d' \in D, r \in R, q \in TR_r$$

$$\sum_{q \in TR_r} q \times RSA_{rqd} \leq \sum_{d \in D} \sum_{q \in TR_r} RSA_{rqd} + M(1 - ESRS_{rd}) \quad (21)$$
$$\forall r \in R, d \in D$$

$$\sum_{q \in TR_r} q \times RSA_{rqd} \geq \sum_{d \in D} \sum_{q \in TR_r} RSA_{rqd} - M(1 - ESRS_{rd}) \quad (22)$$
$$\forall r \in R, d \in D$$

$$\sum_{d \in D} ESRS_{rd} = Z_r \qquad \forall r \in R \quad (23)$$

$$Z_r + ESRS_{rd} + ES_{cs} \leq ESR_{rd}^{cs} + 2 \quad (24)$$
$$\forall r \in R, d \in D, c \in C, s \in S_c, ca_{d|TD_d|}^{c} = 1$$

$$Z_r + ESRS_{rd} + ES_{cs} \geq 3ESR_{rd}^{cs} \quad (25)$$
$$\forall r \in R, d \in D, c \in C, s \in S_c, ca_{d|TD_d|}^{c} = 1$$

$$RSA_{r1d} = SSR_{rd}^{cs} \quad (26)$$
$$\forall r \in R, d \in D, c \in C, s \in S_c, y_s^{c1} = 1, ca_{d1}^{c} = 1$$

$$\sum_{r \in R} \sum_{d \in D} \sum_{\substack{c \in C \\ ca_{d1}^{c}=1, y_s^{c1}=1}} SSR_{rd}^{cs} = \sum_{r \in R} \sum_{d \in D} \sum_{\substack{c \in C \\ ca_{d|TD_d|}^{c}=1}} ESR_{rd}^{cs} \quad (27)$$
$$\forall s \in S_c$$

$$SSR_{rd}^{cs} + ESR_{rd}^{cs} \leq 0 \quad (28)$$
$$\forall r \in R, d \in D, c \in C, s \in S_c, ca_{d|TD_d|}^{c} = 1, y_s^{c1} = 1,$$
$$ca_{d1}^{c} = 1, s \neq S_d$$

$$Dwell^{ct} + Pass^{ct} = 1 \qquad \forall c \in C, t \in TC_c \quad (29)$$

$$RTA^{ct} + l\_dwell^{ct} \times Dwell^{ct} \leq RTD^{ct} \quad (30)$$
$$\forall c \in C, t \in TC_c$$

$$RTA^{ct} \geq RTD^{ct} - M(1 - Pass^{ct}) \quad (31)$$
$$\forall c \in C, t \in TC_c$$

$$RTA^{ct+1} - RTD^{ct} \geq l\_run^{ct} - M(C^{ct}) \quad (32)$$
$$\forall c \in C, t \in \{1,2,...,|TC_c|-1\}, c\_inc_c = 1$$
$$retd^{ct} = -1 \ or \ reta^{ct+1} = -1$$

$$RTA^{ct} - RTD^{c't'} \geq l\_sep^{ct} - M(2 - Track_k^{ct} - Track_k^{c't'}) - M(1 - Pr_c^{c't'} + C^{ct} + C^{c't'}) \quad (33)$$
$$\forall c,c' \in C_{dif}^{sep}, c \neq c', t \in TC_c, t' \in TC_{c'}, dir_c = dir_{c'}$$
$$c\_inc_c = c\_inc_{c'} = 1, st^{c't'} = st^{ct} = s,$$
$$k \in Tr_s^{dir_c}, retd^{c't'} = -1 \ or \ reta^{ct} = -1$$

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$$Pr_c^{c't'} + \sum_{\substack{t\in TC_c \\ s\in S_{c'}, y_s^{ct}=1}} Pr_{c'}^{ct} \geq 1 \quad (34)$$

$$\forall c,c' \in C, c \neq c', t' \in TC_{c'}, dir_c = dir_{c'}, c\_inc_c = c\_inc_{c'} = 1$$

$$(|TC_c| - t + 1)(1 - C^{ct}) \geq (|TC_c| - t + 1) - \sum_{\substack{t'=t \\ y_s^{ct'}=1}}^{|TC_c|} Skip_s^{ct'} - \sum_{t'=t}^{|TC_c|} Pass^{ct'} \quad (35)$$

$$\forall c \in C_{OO}, t \in TC_c, c\_inc_c = c\_inc_{c'} = 1$$

$$(1 - C^{ct}) \leq (|TC_c| - t + 1) - \sum_{\substack{t'=t \\ y_s^{ct'}=1}}^{|TC_c|} Skip_s^{ct'} - \sum_{t'=t}^{|TC_c|} Pass^{ct'} \quad (36)$$

$$\forall c \in C_{OO}, t \in TC_c, c\_inc_c = c\_inc_{c'} = 1, reta^{ct} = -1$$

$$\sum_{t'=t+1}^{|T_c|} C^{ct'} \geq (|T_c| - t)C^{ct} \quad \forall c \in C_{OO}, t \in TC_c \quad (37)$$

$$D^{ct} \geq RTA^{ct} - ota^{ct} - M(C^{ct}) \quad (38)$$

$$\forall c \in C_{OO}, t \in T_c$$

$$t \times ESC^{ct} \leq |TC_c| - \sum_{t\in TC_c} C^{ct} \quad (39)$$

$$\forall c \in C_{OO}, t \in TC_c$$

$$t \times ESC^{ct} \geq |TC_c| - \sum_{t\in T_c} C^{ct} - M(1 - ESC^{ct}) \quad (40)$$

$$\forall c \in C_{OO}, t \in TC_c$$

$$\sum_{t\in TC_c \cup 0} ESC^{ct} = 1 \quad \forall c \in C_{OO} \quad (41)$$

$$DD_c \geq D^{ct} - M(1 - ESC^{ct}) \quad (42)$$

$$\forall c \in C_{OO}, t \in TC_c$$

$$Skip_s^{ct} \geq Pass^{ct} \quad (43)$$

$$\forall c \in C_{OO}, t \in TC_c, y_s^{ct} = 1, act^{ct} = STOP$$

$$Skip_s^{ct} \leq Pass^{ct} \quad (44)$$

$$\forall c \in C_{OO}, t \in TC_c, y_s^{ct} = 1$$

$$\sum_{k\in Tr_s^{dir_c}} Track_k^{ct} = 1 \quad \forall c \in C, t \in T_c, y_s^{ct} = 1, st^{ct} = s \quad (45)$$

$$ES_{cs} = \sum_{t\in TC_c} (y_s^{ct} \times ESC^{ct}) \quad (46)$$

$$\forall c \in C_{OO}, s \in S_c$$

$$RTD^{ct+1} - RTA^{ct} \geq l\_runtime\_1skipi^{ct}(Skip_s^{ct} - Skip_{s'}^{ct+1}) - M(C^{ct}) \quad (47)$$

$$\forall c \in C, t \in TC_c, s, s' \in S_c, y_s^{ct} = 1, y_{s'}^{ct+1} = 1$$

$$RTD^{ct+1} - RTA^{ct} \geq l\_runtime\_1skipj^{ct}(Skip_{s'}^{ct+1} - Skip_s^{ct}) - M(C^{ct}) \quad (48)$$

$$\forall c \in C, t \in TC_c, s, s' \in S_c, y_s^{ct} = 1, y_{s'}^{ct+1} = 1$$

$$RTD^{ct+1} - RTA^{ct} \geq l\_runtime\_2skip^{ct}(Skip_s^{ct} + Skip_{s'}^{ct+1} - 1) - M(C^{ct}) \quad (49)$$

$$\forall c \in C, t \in TC_c, s, s' \in S_c, y_s^{ct} = 1, y_{s'}^{ct+1} = 1$$

$$RTD^{ct+1} - RTA^{ct} \geq l\_runtime\_Nskip^{ct}(1 - Skip_s^{ct} - Skip_{s'}^{ct+1}) - M(C^{ct}) \quad (50)$$

$$\forall c \in C, t \in TC_c, s, s' \in S_c, y_s^{ct} = 1, y_{s'}^{ct+1} = 1$$

$$RTD^{ct} - RTD^{c't'} \geq l\_Nheadway_{c't'}^{ct}(1 - Skip_s^{ct} - Skip_{s'}^{ct+1} - Skip_s^{c't'} - Skip_{s'}^{c't'+1}) - M(1 - Pr_c^{c't'} + C^{ct} + C^{c't'}) \quad (51)$$

$$\forall c,c' \in C_{dif}^{head}, c \neq c', dir_c = dir_{c'}, t = \{1,2,...,|TC_c|-1\}$$
$$t' = \{1,2,...,|TC_{c'}|-1\}, c\_inc_c = c\_inc_{c'} = 1,$$
$$st^{c't'} = st^{ct} = s, st^{c't'+1} = st^{ct+1} = s', retd^{c't'} = -1 \; or \; reta^{ct} = -1$$

$$RTD^{ct} - RTD^{c't'} \geq l\_4headway_{c't'}^{ct}(Skip_s^{ct} + Skip_{s'}^{ct+1} + Skip_s^{c't'} + Skip_{s'}^{c't'+1} - 3) - M(1 - Pr_c^{c't'} + C^{ct} + C^{c't'}) \quad (52)$$

$$\forall c,c' \in C_{dif}^{head}, c \neq c', dir_c = dir_{c'}, t = \{1,2,...,|TC_c|-1\}$$
$$t' = \{1,2,...,|TC_{c'}|-1\}, c\_inc_c = c\_inc_{c'} = 1,$$
$$st^{c't'} = st^{ct} = s, st^{c't'+1} = st^{ct+1} = s', retd^{c't'} = -1 \; or \; reta^{ct} = -1$$

$$RTD^{ct} - RTD^{c't'} \geq l\_1headway\_cs_{c't'}^{ct}(Skip_s^{ct} - Skip_{s'}^{ct+1} - Skip_s^{c't'} - Skip_{s'}^{c't'+1}) - M(1 - Pr_c^{c't'} + C^{ct} + C^{c't'}) \quad (53)$$

$$\forall c,c' \in C_{dif}^{head}, c \neq c', dir_c = dir_{c'}, t = \{1,2,...,|TC_c|-1\}$$
$$t' = \{1,2,...,|TC_{c'}|-1\}, c\_inc_c = c\_inc_{c'} = 1,$$
$$st^{c't'} = st^{ct} = s, st^{c't'+1} = st^{ct+1} = s', retd^{c't'} = -1 \; or \; reta^{ct} = -1$$

$$RTD^{ct} - RTD^{c't'} \geq l\_1headway\_csp_{c't'}^{ct}(Skip_{s'}^{ct+1} - Skip_s^{ct} - Skip_s^{c't'} - Skip_{s'}^{c't'+1}) - M(1 - Pr_c^{c't'} + C^{ct} + C^{c't'}) \quad (54)$$

$$\forall c,c' \in C_{dif}^{head}, c \neq c', dir_c = dir_{c'}, t = \{1,2,...,|TC_c|-1\}$$
$$t' = \{1,2,...,|TC_{c'}|-1\}, c\_inc_c = c\_inc_{c'} = 1,$$
$$st^{c't'} = st^{ct} = s, st^{c't'+1} = st^{ct+1} = s', retd^{c't'} = -1 \; or \; reta^{ct} = -1$$

$$RTD^{ct} - RTD^{c't'} \geq l\_1headway\_cps_{c't'}^{ct}(Skip_s^{c't'} - Skip_{s'}^{ct+1} - Skip_s^{ct} - Skip_{s'}^{c't'+1}) - M(1 - Pr_c^{c't'} + C^{ct} + C^{c't'}) \quad (55)$$

$$\forall c,c' \in C_{dif}^{head}, c \neq c', dir_c = dir_{c'}, t = \{1,2,...,|TC_c|-1\}$$
$$t' = \{1,2,...,|TC_{c'}|-1\}, c\_inc_c = c\_inc_{c'} = 1,$$
$$st^{c't'} = st^{ct} = s, st^{c't'+1} = st^{ct+1} = s', retd^{c't'} = -1 \; or \; reta^{ct} = -1$$

$$RTD^{ct} - RTD^{c't'} \geq l\_1headway\_cpsp_{c't'}^{ct}(Skip_{s'}^{c't'+1} - Skip_s^{c't'} - Skip_{s'}^{ct+1} - Skip_s^{ct}) - M(1 - Pr_c^{c't'} + C^{ct} + C^{c't'}) \quad (56)$$

$$\forall c,c' \in C_{dif}^{head}, c \neq c', dir_c = dir_{c'}, t = \{1,2,...,|TC_c|-1\}$$
$$t' = \{1,2,...,|TC_{c'}|-1\}, c\_inc_c = c\_inc_{c'} = 1,$$
$$st^{c't'} = st^{ct} = s, st^{c't'+1} = st^{ct+1} = s', retd^{c't'} = -1 \; or \; reta^{ct} = -1$$

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$$RTD^{ct}-RTD^{c't'}\geq l\_2headway\_cscsp^{ct}_{c't'}(Skip^{ct}_{s}+Skip^{ct+1}_{s'}-Skip^{c't'+1}_{s'}-Skip^{c't'}_{s}-1)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (57)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_2headway\_cscps^{ct}_{c't'}(Skip^{ct}_{s}+Skip^{c't'}_{s}-Skip^{ct+1}_{s'}-Skip^{c't'+1}_{s'}-1)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (58)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_2headway\_cscpsp^{ct}_{c't'}(Skip^{ct}_{s}+Skip^{c't'+1}_{s'}-Skip^{c't'}_{s}-Skip^{ct+1}_{s'}-1)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (59)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_2headway\_cspcps^{ct}_{c't'}(Skip^{ct+1}_{s'}+Skip^{c't'}_{s}-Skip^{ct}_{s}-Skip^{c't'+1}_{s'}-1)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (60)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_2headway\_cspcpsp^{ct}_{c't'}(Skip^{ct+1}_{s'}+Skip^{c't'+1}_{s'}-Skip^{c't'}_{s}-Skip^{ct}_{s}-1)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (61)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_2headway\_cpscpsp^{ct}_{c't'}(Skip^{c't'}_{s}+Skip^{c't'+1}_{s'}-Skip^{ct+1}_{s'}-Skip^{ct}_{s}-1)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (62)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_3headway\_cs^{ct}_{c't'}(Skip^{c't'}_{s}+Skip^{c't'+1}_{s'}+Skip^{ct+1}_{s'}-Skip^{ct}_{s}-2)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (63)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_3headway\_csp^{ct}_{c't'}(Skip^{c't'}_{s}+Skip^{c't'+1}_{s'}+Skip^{ct}_{s}-Skip^{ct+1}_{s'}-2)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (64)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_3headway\_cps^{ct}_{c't'}(Skip^{c't'+1}_{s'}+Skip^{ct}_{s}+Skip^{ct+1}_{s'}-Skip^{c't'}_{s}-2)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (65)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$RTD^{ct}-RTD^{c't'}\geq l\_3headway\_cpsp^{ct}_{c't'}(Skip^{ct}_{s}+Skip^{ct+1}_{s'}+Skip^{c't'}_{s}-Skip^{c't'+1}_{s'}-2)-M(1-Pr^{c't'}_{c}+C^{ct}+C^{c't'}) \quad (66)$$
$$\forall c,c'\in C^{head}_{dif},c\neq c',dir_c=dir_{c'},t=\{1,2,...,|TC_c|-1\}$$
$$t'=\{1,2,...,|TC_{c'}|-1\},c\_inc_c=c\_inc_{c'}=1,$$
$$st^{c't'}=st^{ct}=s,st^{c't'+1}=st^{ct+1}=s',retd^{c't'}=-1\ or\ reta^{ct}=-1$$

$$STC_{dt+1}-ETC_{dt}\geq 60 \quad (67)$$
$$\forall d\in D,t\in TD_d,ca^{c}_{dt}=1,ca^{c'}_{dt+1}=1,$$
$$dir_c=dir_{c'},se_c\neq PADTLL$$

$$STC_{dt+1}-ETC_{dt}\geq 30 \quad (68)$$
$$\forall d\in D,t\in TD_d,ca^{c}_{dt}=1,ca^{c'}_{dt+1}=1,$$
$$dir_c=dir_{c'},se_c=PADTLL$$

$$STC_{dt+1}-ETC_{dt}\geq 420 \quad (69)$$
$$\forall d\in D,t\in TD_d,ca^{c}_{dt}=1,ca^{c'}_{dt+1}=1,dir_c\neq dir_{c'}$$

$$RTA^{c1}=STC_{dt} \quad (70)$$
$$\forall d\in D,t\in TD_d,c\in C,ca^{c}_{dt}=1$$

$$RTD^{ct}\leq ETC_{dt'} \quad (71)$$
$$\forall d\in D,t'\in TD_d,c\in C,t\in TC_c,ca^{c}_{dt'}=1$$

$$RTD^{ct}\geq ETC_{dt'}-M(1-ESC^{ct}) \quad (72)$$
$$\forall d\in D,t'\in TD_d,c\in C,t\in TC_c,ca^{c}_{dt'}=1$$

$$ETC_{dt}\geq STC_{dt}\quad \forall d\in D,t\in TD_d \quad (73)$$

$$otd^{c1}-RTD^{c1}\leq 300+M(C^{c1}) \quad (74)$$
$$\forall c\in C,retd^{c1}=-1$$

$$RTD^{ct}\geq t\_start\_Erun-M(1-EDrun^{ct}) \quad (75)$$
$$\forall c\in C,t=\{1,2,...,|TC_c|-1\}$$
$$st^{ct}=st\_start\_Erun,st^{ct+1}=st\_end\_Erun$$

$$RTD^{ct}\leq t\_start\_Erun+M(EDrun^{ct}) \quad (76)$$
$$\forall c\in C,t=\{1,2,...,|TC_c|-1\}$$
$$st^{ct}=st\_start\_Erun,st^{ct+1}=st\_end\_Erun$$

$$RTA^{ct}\leq t\_end\_Erun+M(1-EArun^{ct}) \quad (77)$$
$$\forall c\in C,t=\{2,3,...,|TC_c|\}$$
$$st^{ct-1}=st\_start\_Erun,st^{ct}=st\_end\_Erun$$

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$$RTA^{ct} \geq t\_end\_Erun - M\,(EArun^{ct}) \quad (78)$$
$$\forall c \in C, t = \{2,3,...,|TC_c|\}$$
$$st^{ct-1} = st\_start\_Erun, st^{ct} = st\_end\_Erun$$

$$EArun^{ct} + EDrun^{ct+1} \leq Extended\_run^{ct} + 1 \quad (79)$$
$$\forall c \in C, t = \{1,2,...,|TC_c|-1\}$$
$$st^{ct} = st\_start\_Erun, st^{ct+1} = st\_end\_Erun$$

$$EArun^{ct} + EDrun^{ct+1} \geq 2Extended\_run^{ct} \quad (80)$$
$$\forall c \in C, t = \{1,2,...,|TC_c|-1\}$$
$$st^{ct} = st\_start\_Erun, st^{ct+1} = st\_end\_Erun$$

$$RTA^{ct+1} - RTD^{ct} \geq \quad (81)$$
$$l\_Erun - M\,(1 - Extended\_run^{ct} + C^{ct})$$
$$\forall c \in C, t \in \{1,2,...,|TC_c|-1\}, c\_inc_c = 1$$
$$retd^{ct} = -1 \text{ or } reta^{ct+1} = -1$$
$$st^{ct} = st\_start\_Erun, st^{ct+1} = st\_end\_Erun$$

$$RTA^{ct} \geq t\_start\_Edwell - M\,(1 - Edwell^{ct}) \quad (82)$$
$$\forall c \in C, t \in TC_c, st^{ct} = st\_Edwell$$

$$RTA^{ct} \leq t\_start\_Edwell + M\,(Edwell^{ct}) \quad (83)$$
$$\forall c \in C, t \in TC_c, st^{ct} = st\_Edwell$$

$$RTA^{ct} \leq t\_end\_Edwell + M\,(1 - Edwellp^{ct}) \quad (84)$$
$$\forall c \in C, t \in TC_c, st^{ct} = st\_Edwell$$

$$RTA^{ct} \geq t\_end\_Edwell - M\,(Edwellp^{ct}) \quad (85)$$
$$\forall c \in C, t \in TC_c, st^{ct} = st\_Edwell$$

$$Edwellp^{ct} + Edwell^{ct} \leq Extended\_dwell^{ct} + 1 \quad (86)$$
$$\forall c \in C, t \in TC_c, st^{ct} = st\_Edwell$$

$$Edwellp^{ct} + Edwell^{ct} \geq 2Extended\_dwell^{ct} \quad (87)$$
$$\forall c \in C, t \in TC_c, st^{ct} = st\_Edwell$$

$$RTD^{ct} - RTA^{ct} \geq l\_Edwell \times Dwell^{ct} \quad (88)$$
$$-M\,(1 - Extended\_dwell^{ct})$$
$$\forall c \in C, t \in TC_c, st^{ct} = st\_Edwell$$

$$RTA^{ct}, RTD^{ct} \geq 0 \qquad \forall c \in C, t \in TC_c \quad (89)$$

$$D^{ct} \geq 0 \qquad \forall c \in C_{OO}, t \in TC_c \quad (90)$$

$$DD_c, Pen\_D_c \geq 0 \qquad \forall c \in C_{OO} \quad (91)$$

$$Pen\_PF_{c't'}^{ct} \geq 0 \quad (92)$$
$$\forall c, c' \in C, c \neq c', t \in TC_c, t' \in TC_{c'},$$
$$(c,t),(c',t') \in Seq_{EB} \text{ or } (c,t),(c',t') \in Seq_{WB}$$

$$STC_{dt}, ETC_{dt} \geq 0 \qquad \forall d \in D, t \in TD_d \quad (93)$$

$$Delay_c \in \{0,1\} \qquad \forall c \in C_{OO} \quad (94)$$

$$Y\_PF_{c't'}^{ct} \in \{0,1\} \quad (95)$$
$$\forall c, c' \in C, c \neq c', t \in TC_c, t' \in TC_{c'},$$
$$(c,t),(c',t') \in Seq_{EB} \text{ or } (c,t),(c',t') \in Seq_{WB}$$

$$C^{ct}, ESC^{ct}, Dwell^{ct}, Pass^{ct} \in \{0,1\} \quad (96)$$
$$\forall c \in C, t \in TC_c$$

$$ESR_{rd}^{cs}, SSR_{rd}^{cs} \in \{0,1\} \quad (97)$$
$$\forall r \in R, d \in D, c \in C, s \in S_c$$

$$EDrun^{ct}, EArun^{ct}, Extended\_run^{ct} \in \{0,1\} \quad (98)$$
$$\forall c \in C, t \in TC_c$$

$$Edwell^{ct}, Edwellp^{ct}, Extended\_dwell^{ct} \in \{0,1\} \quad (99)$$
$$\forall c \in C, t \in TC_c$$

$$RSA_{rqd} \in \{0,1\} \qquad \forall r \in R, q \in TR_r, d \in D \quad (100)$$

$$Z_r \in \{0,1\} \qquad \forall r \in R \quad (101)$$

$$ES_{cs} \in \{0,1\} \qquad \forall c \in C, s \in S_c \quad (102)$$

$$ESRS_{rd} \in \{0,1\} \qquad \forall r \in R, d \in D \quad (103)$$

$$Track_k^{ct} \in \{0,1\} \quad (104)$$
$$\forall c \in C, t \in TC_c, y_s^{ct} = 1, st^{ct} = s, k \in Tr_s^{dir_c}$$

$$Pr_c^{c't'} \in \{0,1\} \quad (105)$$
$$\forall c, c' \in C, c \neq c', t' \in TC_{c'}, dir_c = dir_{c'},$$
$$c\_inc_c = c\_inc_{c'} = 1$$

$$Skip_s^{ct} \in \{0,1\} \quad (106)$$
$$\forall c \in C_{OO}, t \in TC_c, y_s^{ct} = 1, act^{ct} = STOP$$

The objective function (1) consists of three parts. Depending on the station and time, there will be a penalty for each planned stop skipped in the amended timetable for courses in category OO. The first part is related to this penalty. To avoid the nonlinearity of the model, the value of the parameter ssf_s^{ct} has been set to 1. Also, an economic penalty will be applied to every course in category OO if it is delayed at the last station of its actual journey. The second part is related to this penalty. Passengers face a loss of service quality in the central section of the network when train passage frequency is lower than a given

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threshold. As a result, the third part represents the costs of passage frequency penalties at two reference stations (PADTLL for WB and WCHAPXR for EB).

By constraints (2) to (5), if the arrival delay time of course *c* at the last station of its actual journey is more than 3 minutes, the amount of arrival delay time is calculated for the objective function penalty. The sequence of course arrivals at two reference stations, *PADTLL* for *WB* courses and *WCHAPXR* for *EB* courses, is calculated using constraints (6) to (11). The passage frequency penalties are calculated with constraints (12) and (13). A reference station is used to determine passage frequency based on the headway between successive arrivals of revenue services (PADTLL for WB courses and WCHAPXR for EB courses). $ht_{c't'}^{ct}$ is a threshold headway for pair $(ct, c't')$ and depends on the planned arrival times ct and $c't'$.

Constraint (14) ensures that every duty must be assigned to one sequence of an active rolling stock. Constraint (15) guarantees that one duty can be assigned to each sequence of an active rolling stock. According to constraint (16), if rolling stock *r* is used, the variable Z_r takes the value 1; otherwise, it takes the value 0. Constraint (17) ensures that the solution cannot use more than 76 train sets. Constraint (18) ensures that the end station of one duty set and the start station of the following duty set for the same rolling stock are the same.

Constraint (19) places the duties consecutively in active rolling stock sequences. Therefore, if no duty is assigned to a sequence of rolling stock, the subsequent sequences must remain empty. According to the problem description document, a 420-second minimum connection time will be imposed between train sets. It is required that a rolling stock waits at least 7 minutes before being used for another duty after completing its duties at a node.

By constraint (20), the minimum connection time between the end of one duty set and the start of the next for the same rolling stock is observed.

The last duty assigned to a rolling stock is determined using constraints (21) to (23). Constraints (24) and (25) determine the last station where a rolling stock completes its last set of duties on its actual journey. Similarly, constraint (26) specifies the station where the rolling stock starts its journey. The rolling stock balance is guaranteed by constraint (27). Accordingly, the number of duties ending at a node must equal the number originating at that node. It is mandatory to enforce the start and stop of a rolling stock duty at one of the depot stations (*GIDEPKS*, *OLDOXRS*, *SHENFMS*, *PLMSXCR*, and *MDNHCHS*). Constraint (28) is written for this purpose.

A train can cross each node with two different activities, i.e., passing or stopping. The activity of each course sequence is determined by constraint (29). The stations that trains pass through would have equal arrival and departure times. The train stops at stations will have a positive dwell time according to the planned timetable. With constraints (30) and (31), the difference between the arrival time and departure time of a station is calculated.

According to the distance between the two stations, a minimum time between leaving the first station and entering the second station is considered by constraint (32). The value of this minimum duration is calculated from the planned timetable. Also, a minimum separation time between the end of the occupation of a track by a train and the beginning of the occupation of the same track by the following one l_sep^{ct}, provided that each track can be used by at most one train at a time. Constraint (33) is written for this purpose. Constraint (34) specifies the sequence of courses with the same direction at each station. With constraints (35) to (38), the course sequences that have been canceled are specified. The arrival delay time of sequence *t* corresponding to course *c* is calculated by constraint (38). Constraints (38) to (41) determine the last uncancelled sequence for each course. The arrival delay time of each course at the last station of its actual journey is calculated by constraint (42). A skipped stop occurs when a train passes through a station instead of stopping as planned. The skipped sequences are determined by constraints (43) and (44). For each node, a set of tracks is defined for each direction. Constraint (45) ensures that each course sequence is assigned to a track from the station tracks and corresponds to the direction of the course. The last uncancelled station of each course is calculated by constraint (46).

The minimum run time between two adjacent stations depends on the activities at the

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corresponding stations, i.e., pass/stop, pass/pass, stop/stop, and stop/pass. Furthermore, there is a minimum technical run time depending on the course's direction and the pair of activities in the nodes. Constraints (47) to (50) guarantee the minimum run time considering the actual activities of each course sequence. For example, constraint (47) is activated when the planned stop is skipped at the first station of the edge. Also, constraint (49) is activated when the planned stop is skipped at both stations.

There is also a minimum head-to-head headway between the entrance times of successive trains traveling in the same direction. This minimum headway time depends on the activity pair of each train in the stations. The line headway constraints set a minimum time separation between a couple of trains using the same edge in the same direction. This separation is calculated between the departure times of the two trains from the first node of the edge, according to their direction. Considering that the minimum headway time value depends on each node's actual activity, 16 different constraints are presented in constraints (51) to (66) for this purpose. For example, constraint (51) is activated when trains stop at stations as in the planned timetable, and the planned stop is not ignored. As another example, constraint (53) is activated when the front train skips the planned stop at the first station of the edge, but the behind train moves from two stations according to the planned timetable. Other constraints were also written with this logic.

Terminal operations require a minimum technical time, called the CHANGE_END. It typically spans 60 seconds (without cabin turn-over) to 420 seconds (with cabin turn-over). The RAS2022 Problem Solving Team specified the following times:

1) CHNGE_END between trains in the same direction at *PADTLL*: 30s
2) CHNGE_END between trains in the same direction at all the other stations: 60s
3) CHNGE_END between trains with different directions at all stations: 420s

According to these assumptions, constraints (67) to (69) were written to guarantee the CHANGE_END time. Constraint (70) indicates that if a course is assigned to a sequence of one duty, the start time of the course and that sequence of duty must be equal. Constraints (71) to (72) state that if a course is assigned to a sequence of one duty, its departure time at its stations is less than the end time of that sequence of duty, and they are equal at the last station of the course. According to constraint (73), the start time of a duty sequence must be less than the end time of that sequence. Amended timetables cannot be scheduled earlier than 5 minutes from the originally planned times, and constraint (74) ensures this issue.

Constraints (75) to (88) apply the effects of incidents in the model. Constraints (75) to (81) are related to the incident of "extended running times between two adjacent stations." This incident imposes an extended run time significantly more than planned on all trains passing through between two adjacent stations (direction given by the start/end node) within a given time band. This extension is independent of rolling stock. Constraints (75) to (80) specify the course sequences that are located in the time band of this incident at the start/end nodes, and using constraint (81), the increase in run time is considered. Constraints (82) to (88) are related to the incident of "extended dwell time for all trains that stop at a station." This incident imposes that all courses that stop at a station within a time band will be subject to dwell times significantly longer than scheduled.

Similarly, constraints (82) to (87) specify the course sequences that are located in the time band of this incident at the incident station, and using constraint (88), the increase in dwell time is considered. It is noteworthy that if two incidents, "late departure from depot" and "extended dwell time for a train at a station" happen, their effects will be applied to the model's parameters and decision variables. According to this, in the incident of late departure, the rescheduled departure time from the depot (RTD^{c1}) will be fixed based on the departure delay. Also, the parameter value l_dwell^{ct} is updated in the extended train dwell time incident. Finally, constraints (89) to (106) indicate the domain of decision variables.

4. Computational results

Three datasets published by the RAS2022 competition [3] are solved to investigate the model's performance in designing real-time traffic management for an urban rail transit system. The proposed model was implemented in the Visual Studio C# 2019 environment and

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solved by calling Gurobi 9.5.2 solver on a computer system with a Common KVM processor of 2.39 GHz (8 processors) and 64 GB of RAM in a Windows 10 environment. This section overviews the problem instances and summarizes the numerical results.

4.1. Problem instances

We tested our model on London's new Elizabeth line, UK. The Elizabeth Line is an east-west traffic corridor with over 100 kilometers, 104 nodes, and 1259 planned train services in a time window of [0, 86400] s. The Elizabeth Crossrail line connects Reading and Heathrow in the west with Shenfield and Abbeywood in the east through the central tunnels. Finally, it will provide a transport service between the east to west of the British capital, with several connections to the railway and underground networks and the Heathrow international airport. It is important to note that the rail network is shaped like a "horizontal X," with four terminals at the extremities: Abbey Wood and Shenfield on the east and Reading and Heathrow Airport on the west. From/at these terminals and intermediate stations, planned services will originate/terminate. An illustration of the network is provided in Figure 1**Error! Reference source not found.**.

There are several types of nodes within a rail network, but the most important are the stations, junctions, halts, and control points at a macroscopic level. The category field was provided as information for the node. Train stations are places where trains can pick up or drop off passengers at a particular time. There are 217 edges that connect adjacent nodes. In addition, this line has 28 tracks. In the planning timetable, 1259 courses with the category of EE (empty/non-revenue ride) or OO (passenger service) are assigned to 94 duties. There is a minimum headway (time between successive trains in the same direction) of 150 seconds in the central part of the network, where all services between different destinations and origins travel the same tracks. A Communication-Based Control (CBC) system in this network section will permit a nominal minimum technical headway of about 90 seconds. A conventional signaling system in the peripheric branch will permit an approximate minimum technical headway of 240 seconds.

Three evaluation problem instances were provided in this study, which were used to test and evaluate the proposed model. The description of these three instances is as follows:

1. **Incident_ABWDXR:** Timetable snapshot at 6:15 PM: Trains depart from Abbey Wood terminal station with significant delays. This delay will continue until approximately 6:30 PM with a LATE_DEPARTURE incident.
2. **Incident_to_Heathrow:** Timetable snapshot at 8:50 AM: Trains heading to Heathrow Airport are running late; moreover, an "EXTENDED RUN TIME" incident is forecasted until 9:25 AM, imposing an 8 min run time (instead of the planned 3 min) on edge between HTRWTJN and HTRWAPT.
3. **Incident_inside_COS:** Timetable snapshot at 8:00 AM: Traffic inside the central part of the network is perturbed, with an EXTENDED_STATION_DWELL incident at Whitechapel between 8:00 AM and 9:00 AM.

4.2. Numerical results

The MDRTM was implemented with a run-time limit of 30000 seconds, considering

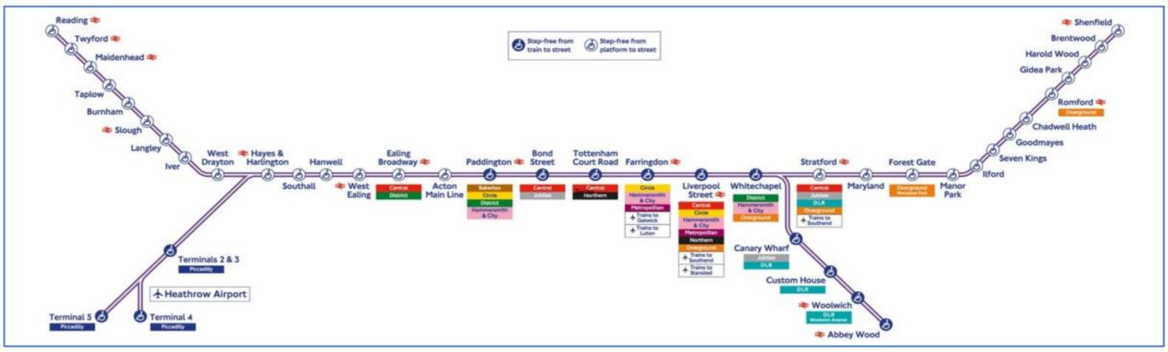

Figure 1:The Elizabeth line

constraints (1) to (106). For memory management, in the process of solving the model, constraints (14) to (66) and (75) to (88) were defined as lazy constraints in the Gurobi solver. The results are reported in Table 4. In addition to each instance's objective function, the percentage of the Gurobi gap (%) and Gurobi solution time (seconds) are also specified in this table. In addition, columns "Skipped stops," "Destination delays," and "Passage frequency" show the value of each part of the objective function for the best-found solution for each problem instance.

Table 4: Results of the MDRTM model

No. Instance	**1**	**2**	**3**
Obj. function	43751	383655	54175
Skipped stops penalty	3626	4280	12300
Destination delays penalty	40125	379375	41875
Passage frequency penalty	0	0	0
Gurobi Gap (%)	0	0	2.00
Gurobi Solution Time (s)	1697	6759	30000
No. of delayed courses	1	5	1
No. of canceled courses	0	0	0
% of skipped sequences	3.95	2.63	2.88

As shown in Table 4, the MDRTM model can solve all three evaluation problem instances with good quality within the considered time limit.

This model has reached the optimal solution for Incident_ABWDXR at a suitable time. As stated in column "Number of delayed courses," in this instance, only one course (9T68RN#1) is delayed, and other courses are completed according to the planning timetable. Also, only 3.95% of the planned stops were skipped. The next noteworthy point is that no course has been canceled.

This model has reached the optimal solution for Incident_to_Heathrow at a suitable time. In this instance, no course was canceled, and only five courses (9N42RL#1, 9R38RL#1, 9R44RL#1, 9T46RL#1, and 9Y50RL#1) arrived at their destination late. Also, only 2.63% of the planned stops were skipped.

The model found a feasible solution for Incident_inside_COS with a relative Gurobi gap percentage of 2.00. In this instance, no course was canceled, and only one course (9Y41RL#1) arrived at its destination late. Also, only 2.88% of the planned stops were skipped.

It should be mentioned that the passage frequency penalties were equal to zero in all instances. In addition, the delay was used to return to planning without canceling any courses in all three instances. Consequently, the MDRTM model seems very useful for designing real-time traffic management for an urban rail transit system based on the obtained results.

5. Conclusions

This paper proposes a MILP model for rescheduling a timetable during railway disruptions, where flexible stopping is innovatively integrated with delaying, canceling, and re-ordering. Flexible stopping means that the originally scheduled stops could be skipped for each train. In contrast, additional stops could be added, considering that during disruptions, a skipped stop could reduce the delays of passengers at their foreseen destinations. Conversley, an added stop could provide passengers with more alternative paths for re-routing.

A real-world instance of London's new Elizabeth line, UK, was used to test the proposed model based on several disruption scenarios. The MDRTM model can solve all three evaluation problem instances with good quality within the time limit. According to our experimental results, a delay is preferable to course cancellation, and exact information on the duration of disruption is beneficial for reducing the total weighted train delay in the future. In real life, the duration of disruption is uncertain. Thus, future direction extends the model to deal with uncertain disruption duration.

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