



Design, Development and Test of a Practical Train Energy Optimization using GA-PSO Algorithm

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<p>Article history: Received: 29.02.2017 Accepted: 25.04.2017 Published: 17.06.2017</p> <hr style="width: 20%; margin-left: 0;"/> <p>Keywords: Optimal speed profile Tractive effort minimization GA-PSO algorithm Genetic algorithm PSO algorithm</p>	<p>One of the strategies for reduction of energy consumption in railway systems is to execute efficient driving by presenting optimized speed profile considering running time, energy consumption and practical constraints. In this paper, by using real route data, an approach based on combination of Genetic and Particle swarm (GA-PSO) algorithms in order to optimize the fuel consumption is provided. The model of train takes into account the length and mass of train, running resistance, tractive effort curves for each notch, signaling system, variations of the motor efficiency with respect to speed and effort ratio, auxiliary equipment consumption and rotary inertia. The route characteristics included in the model are speed limits, gradients, gradient transitions (and its effect along the train) and curves. GA-PSO algorithm combining the benefits of both the original algorithms GA and PSO is validated by formulating the optimization problem. The GA-PSO performance is evaluated by comparing it with a GA algorithm. Further, it is used for obtaining the optimal speed profiles for a locomotive equipped with a GT26CW engine on Tehran-Tappe_sefid block.</p>

1. Introduction

The trains on most railway systems are driven based on the drivers' knowledge of the line, current train parameters, and of course, on the information they obtain through the signaling system. The basic idea of this research is to give the drivers some information from the route and advises on the way of driving, thereby enhancing their ability to run the trains in an optimal way. GA and PSO algorithms both have their own advantages and disadvantages. Based on the results of this research their combination can improve their performances. In this article, speed profiles are produced offline.

Howlett and P. J. Pudney [1, 2] and Chang. C. S & Sim. S. S [3] proposed GA as an optimal approach and they have studied the train optimal control. Wei et al. in [4] have introduced an approach for optimization problem based on variable-length real matrix coding

Multi-Population Genetic Algorithm (MPGA) is proposed. This method is claimed to be able to optimize train energy consumption, improve convergence rate, and promote stability. In [5] by considering movement constraint, a model of urban transit network based on genetic-annealing algorithm, to optimize time and energy consumption on three lines is proposed. The authors claim that implementation of this method leads to energy-saving ratio above 20%. Chen et al. [6] proposed genetic algorithm to be used to optimize train scheduling, avoiding the simultaneous acceleration of too many trains, in order to reduce peak power consumption.

In [7] by developing an optimization model for maximizing the utilization of regenerative energy through modifying dwell time for trains of stations. In this reference, a hybrid genetic/linear programming algorithm was implemented to solve this problem. Bocharnikov et al. [8] presented a

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single train speed profile optimized model by genetic algorithm, considering both tractive energy consumption and utilization of regenerative energy. In addition, a multi-train simulation for estimating the effects of the optimal speed profile on reducing energy consumption was performed. Ding et al. [9] have designed a genetic algorithm to search for the optimal solution by formulating the energy-efficient train operation problem as a two-level optimization model. The first level defines the suitable coasting point of running section of the trains, and the second level determines the travel time for each section to minimize the tractive energy consumption. Han et al. [10] considered a train that is driven by automatic operation mode along a non-constant gradient, curve and with speed limits. Genetic algorithms used to solve the optimal train driving strategy. The approaches based on simulation offer more promising alternatives. They do not require simplifications in the models and enable an accurate calculation of the running times and the energy consumption.

Wang et al. proposed a train trajectory optimization method under real-time rail traffic management in which a set of positions with a target time and speed point is considered for each train. With objective of minimization the energy consumption, this problem is formulated as a multiple-phase optimization model [11]. Zhao developed a train trajectory optimization algorithm in which a tradeoff between reductions in train energy usage and increases in delay penalty has been considered. Three methods includes enhanced brute force, ant colony optimization, and genetic algorithm, are used to find the optimal results efficiently [12]. Shi et al. decomposed the train operational plan into two sub-problems, i.e., the train departure profile optimization which optimizes frequency setting, timetabling and the rolling stock circulation at the terminal without a yard and the rolling stock circulation optimization in which maximum headway function is generated to ensure the service of the train operational plan without considering travel demand [13].

Haahr et al. proposed a time-space graph formulation which can be solved by dynamic programming. In this algorithm, instead of using uniform discretization of time and space, an event-based decomposition which reduces the search space is used. This approach has good property such as high flexibility, easy to handle, e.g. speed limits, changes in altitude, and passage points that need to be crossed within a given time window [14]. Wang et al. proposed the greedy approach and the simultaneous approach to solve optimal train trajectory. The trajectory planning problem is

transformed into a mixed integer linear programming problem. To formulate the problem, the varying line resistance, variable speed restrictions, and maximum traction force are included in the main problem. A piece-wise affine model is considered as the nonlinear train model and the energy consumption of trains is considered as the objective function [15].

Scheepmaker et al. presented a literature review on energy-efficient train control and the related topic of energy-efficient train timetabling, from the first simple models from the 1960s of a train running on a level track to the advanced models and algorithms of the last decade [16]. Meanwhile, Pontryagin's Maximum Principle has been detailed to derive optimal driving strategy of a train under different conditions. Ye et al. addressed the optimal control of a fleet of interacting trains, and the optimal train control involving scheduling. Two method is proposed to solve the problem. The first assumes an operation sequence of maximum traction effort, speed holding, coasting, and maximum braking on each subsection of the track. The travel distance and energy consumption of each operation can be calculated in a closed-form and the optimal train control problem is formulated and solved as a nonlinear programming problem [17].

Kamuzava et al. developed an energy-efficient speed profile generator by combining partial energy-oriented driving approaches when the planned running time is given. In this method, the generator has been added to existing shortest running time calculation software, so that it works for various vehicle and train route data [18]. Su et al. used a parallel multi population genetic algorithm to optimize the train control strategy, which considers not only energy consumption, but also running time, security, and riding comfort. In this method, train traction property and braking property was detailed to ensure the accuracy of running [19].

NSGA-II [20] is a variant of NSGA (Non-dominated Sorting Genetic Algorithm). Evolutionary algorithms are population-based algorithms that possess an in-built mechanism to explore the different parts of the Pareto front simultaneously.

2. Train Simulation Model

For designing an energy efficient driving profile, the decision variables are speed (v) and

control settings (j) in order to minimize the cost function (J) by considering the train journey constraints (speed limitation, maximum traction effort, etc.) to make accurate decision. The model of train takes into account the length and mass of train, running resistance, traction and brake force curves, variation of motor efficiency in each control setting with respect to speed and effort ratio. The line characteristics involved in the model are speed limits, tunnels, gradients, grade transitions, bends and their effects along the train.

For locomotives with discrete control settings, such as GT26-CW Diesel-electric locomotive, the control mechanism of locomotive can be represented by an integer control variable j . Each non-negative value of control variable determines a traction control and each negative integer value determines a brake control. It is assumed that traction force $f_j > 0$ when $j > 0$ and $f_j < 0$ when $j < 0$, the system is in coasting mode. When $j \geq 0$ the power generated by the diesel is directly proportional to the rate of fuel consumption. When $j < 0$, a constant, negative force applied to train [2], when the train is in braking mode.

Train operation state equation can be described as follows:

$$v \frac{dv}{dx} = f_j(v) - w_0(v) - b_j(v) - f_g(x) \quad (1)$$

$$\frac{dt}{dv} = \frac{1}{v} \quad (2)$$

where v, t, x are respectively train speed, operation time and train position, j is control setting and $f_j(v), w_0(v), f_g(x), b_j(v)$ are traction, basic resistance, gradient resistance and braking force applied as per unit mass. Train departs from starting point and arrives at station during the determined time T . Train movement is constrained by Eq. (3):

$$v(x) \leq V_{\text{lim}}(x) \quad (3)$$

where $V_{\text{lim}}(x)$ is speed limit. The cost function is defined as:

$$J = \int_0^x f_j(v) dx + (x(T) - X) \quad (4)$$

where X is the final destination of train movement. The cost function is constructed of two sections, the integral of traction force and distance of the train to the destination at time T . The goal is to minimize these two sections.

3. Optimal Speed Profile Designers

3.1. Genetic algorithm

GA is an iterative heuristic algorithm that has a stochastic search mechanism to find optimized solution. This algorithm uses techniques such as inheritance, mutation, selection and crossover to optimize the problem. In this sub section, genetic algorithm is designed to solve the integrated energy-efficient operation model. The algorithm has the following steps.

1- GA starts by randomly generating an initial population of solutions [21]. Three variables are selected as notch, lower speed and higher speed for each accelerating, coasting and Speed Holding process. The lower and upper bond of each variable is placed in a vector.

2- Four individuals as Positions, Cost, Best Position and Best Cost is defined. In position individual data, three variables are produced and solutions are ranked according to the cost function and then Best Position and Best Cost for each iteration.

3- Figure 1 presents general process of GA algorithm. The algorithm process can be summarized as follows:

- Selection operator: This operator selects a number of chromosomes in a population. It is obvious that the more fitted chromosomes have higher chance to be selected for breeding.

- Crossover operator: This operator applies on a chromosome pair and generates a new chromosome pair. Usually the possibility of crossover for each pair of chromosomes is 0.6 to 0.95 [22].

- Mutation operator: At the end of Crossover operation, Mutation operator applies on chromosomes. This operator selects randomly a gene on chromosome and alters the gene content.

- By using Crossover and Mutation parents are selected for breeding and are combined together to produce new offspring. This

process is repeated several times to produce the next generation population.

- The genetic operation runs until termination condition are achieved.

4- Finally, the program returns to the second step and processes repeat [21].

3.2. Particle swarm algorithm

The PSO algorithm conducts a search using population of individuals. The individual in the population is called the particle and the population is called the swarm [23, 24]. In PSO, each particle has a velocity, which is initialized randomly based upon the bounds prescribed. In addition to the velocity, PSO algorithm has a memory of local best (pbest) and global best values (gbest). Particle's new velocity is updated based on its previous velocity, position, pbest, and gbest positions Eq. (5), in Eq. (6) the new position is calculated by new velocity [25].

$$v_i(n) = wv_i(n-1) + c_1r_1(p_i - x_i(n-1)) + c_2r_2(g_i - x_i(n-1)) \quad (5)$$

$$x_i(n) = x_i(n-1) + v_i(n) \quad (6)$$

Where $i = 1, 2, \dots, N_p; n = 1, 2, \dots, N$. N_p is the size of swarm and N is iteration limit; c_1 and c_2 are positive constants that are called "social factors"; r_1 and r_2 are random numbers between 0 and 1; w determines the impact of previous history of velocities on the current one. p_i is i th population, x_i is position of i th population and g_i is the best position.

3.3. GA-PSO algorithm

GA has a better diversity control during the initial stages of the search process, which results in slow convergence, whereas PSO has faster convergence rate. These two algorithms have both the features to make an ideal search algorithm. Table 1 shows a comparison of two algorithms and presents the advantages and disadvantages of each algorithm.

A flowchart of a GA-PSO algorithm is shown in Figure 2. Since the new generation of population in GA is created by eliminating the old generation, the total population is sorted and the better half is used to create new population.

Before updating the velocity of particles, the function values of the GA population is used to update pbest and gbest directly. Instead of updating the gbest and pbest once for every generation in PSO, GA populations coupling helps to update the pbest and gbest values. During this step, the velocity of the particles is not updated. This additional step of updating the local, pbest, and global, gbest, best values show considerable improvement in speeding up the algorithm. . GA and PSO algorithms are both found to have the features desired in an ideal optimization algorithm. However, each method has a limitation, it is important to have both the contrasting features robustness and the speed of convergence in the optimization algorithm to be used for engineering applications. The control parameters of both GA and PSO can be tuned to achieve robustness or better speed of convergence. In this article, the speed of convergence is not considered because the speed profiles are produced offline.

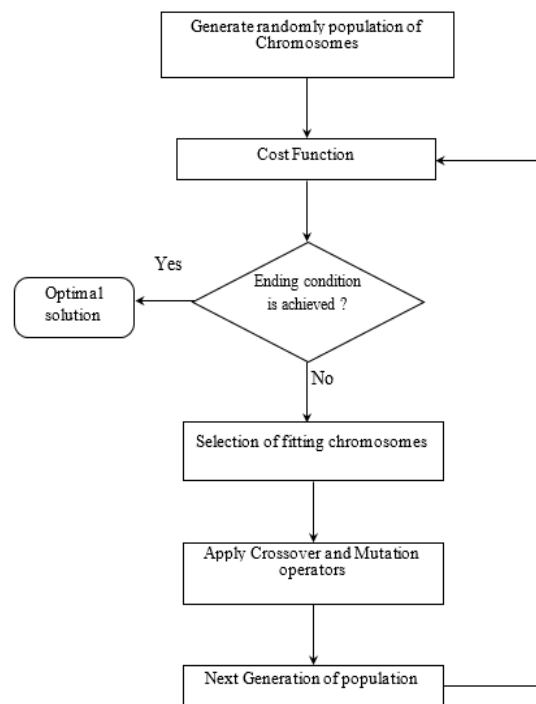


Figure 1. Flowchart of GA process

4. Measurement and Calculation of Train Resistance

Different railways have used various approaches for calculating the train and track resistances [26-27].The main purpose of investigating a method to measure and calculate train main resistances is providing accurate information on elements that effect train energy

consumption. In this paper, direct method is used to calculate Davis formula according to Eq. (1).

Table 1. Advantages and disadvantages of GA and PSO algorithms

disadvantages	advantages	Algorithm
-By increasing the dimension, efficiency of the algorithm is reduced	-Built-in memory that keeps the best velocity of previous history.	PSO
-The answer is local	-Easy implementation, -Low number of parameters	
-Slow convergence rate,	-global search space, Problem independence,	GA [24]
-More CPU time	-robustness of search	

4.1. Process of modeling train running resistance

Train tractive effort is calculated by GT26-CW locomotive generator power through current and voltage sensors data and considering efficiency of traction motor. We need to have accurate data of track gradients and speed of train. Therefore, all calculations are done in approximately zero acceleration or holding speed. Otherwise, train acceleration should be considered.

Prior to calculate train resistance, we need information of train speed, track gradients, tractive effort, train mass, the number of wagons, some geometrical parameters of the train like train area, train length, number of locomotive and wagons axels. The effects of resistance due to track curvature are ignored in current calculation because except for every tight curves of less than about 250 m radius, the effects are small [28].

Train resistance is obtained by using a force balance. The total running resistance is represented by Eq. (7):

$$w_0(v) = f_j(v) - v \frac{dv}{dx} - f_g(x) \quad (7)$$

The calculation is done on routes that brake force is zero. $v \frac{dv}{dx}$ is the term that is called total force per unit mass. To obtain $w_0(v)$ we require the traction force, the gradient force and the total force. Assuming the train is considered as a point mass, the gradient force per unit mass can be calculated by:

$$f_g(x) = g \sin(\alpha(x)) \quad (8)$$

where g is the gravitational constant and α is the angle of the gradient, which is positive when the train is running uphill.

The information of train for 100 routes are analyzed and for every part of track, train resistance is calculated and curve-fitting method is used to extract the train main resistance in Eq. (9). Resistance force in kgf/ton and speed in kilometers per hour and a comparison between train resistance forces that is calculated and the UIC, SNCF and Shin Kasen's Series 100 [28] is presented in Figure 3.

$$R = 2.148 + 0.002V + 0.0002V^2 \quad (9)$$

5. Tractive Effort Modeling

When designing a speed profile to guide the train driver to follow, it is necessary to know how much tractive effort is utilized in every notch. The tractive effort of a Diesel-electric locomotive can be assessed by analyzing the locomotive generator power through the empirical formula (10).

$$F_t = \frac{\eta P_G}{V} \times 360 \quad (10)$$

where P_G is generator power in kW, V is the velocity in km/h, η is the generator efficiency, and F_t is tractive effort of a Diesel-electric locomotive in N, Tractive effort of GT26CW locomotive, measured at each notch is shown in Figure 4.

Based on the real data, tractive effort can be estimated by interpolation method. For example, Figure 5 represents an estimated tractive effort for notch 8.

Train parameters are introduced in Table 2. Also Table 3 shows the tuned parameters of GA-PSO algorithm.

As shown in Figure 7, after block signal, speed increases and accordingly control setting increases up to notch 8. Therefore, by increasing in speed notch increases and due to the descending path, most of the block is passed by idle notch and speed profile is in coasting mode.

6. Simulation and Results

To illustrate the effectiveness of the proposed model and algorithm, a case study based on the Tehran- Tappe_sefid line (Figure 6) is represented. Tehran- Tappe_sefid line has a length of 10.1 km,

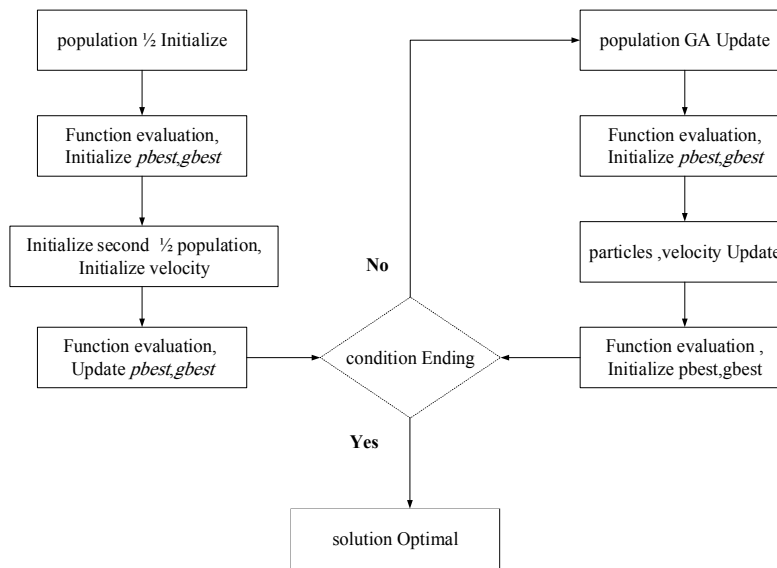


Figure 2. Flow chart of GA-PSO algorithm

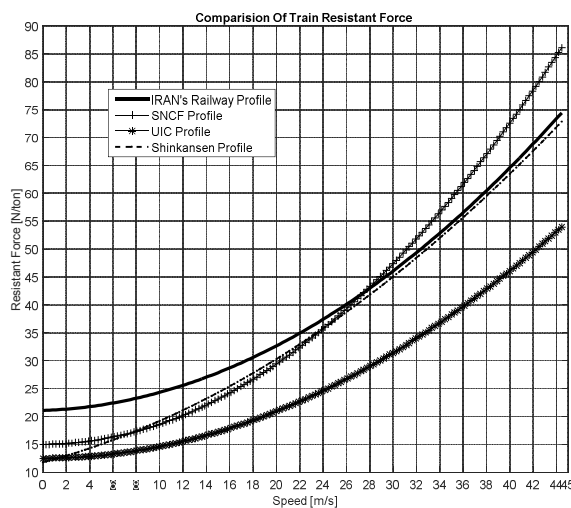


Figure 3. Comparison of the train resistance forces

consisting of six blocks, with an average gradient of -1.4 in 1000 meter. The maximum line speed is defined to be 120 km/h.

Figure 7, describes the optimization for the speed profile produced by GA-PSO algorithm for Tehran- Tappe_sfid block. The results shown are accompanied by notch for each moment, between the block and distance signals. Table 4 shows the results of simulation.

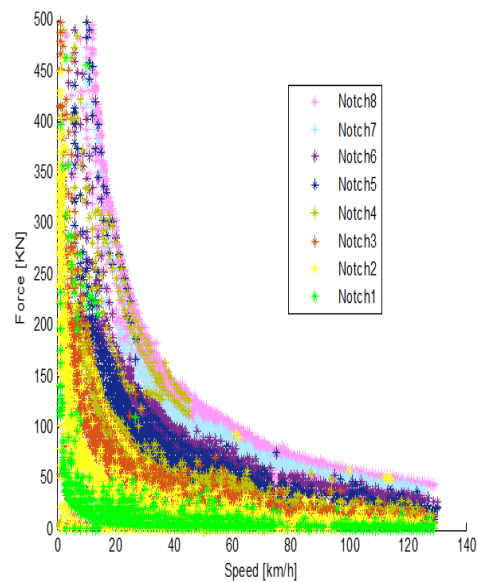


Figure 4. Tractive effort at each notch

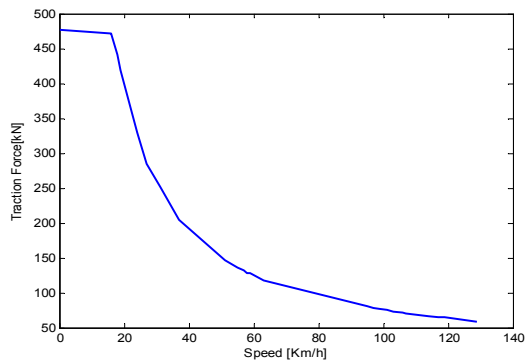


Figure 5. Tractive force at notch 8

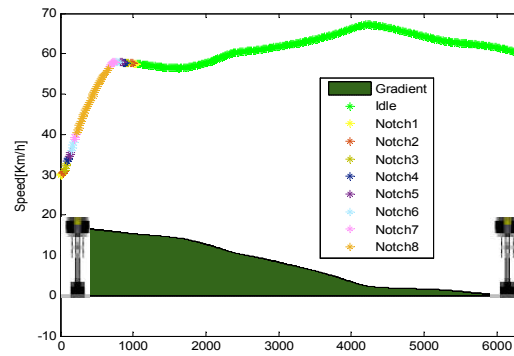


Figure 7. GA-PSO optimal speed profile.

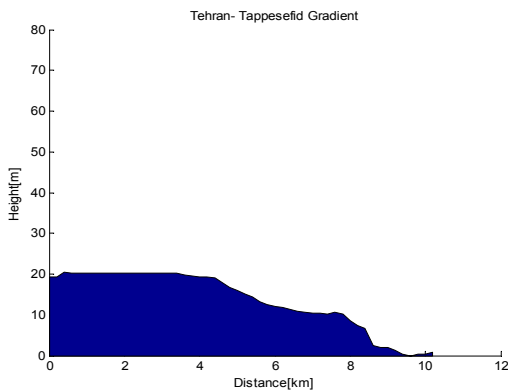


Figure 6. Tehran- Tappe_sefid gradient

Table 4. GA-PSO simulation results

Total energy consumption (MJ)	76.18
Fuel consumption (liter)	9.50
Trip Time (min)	6.5
Distance (km)	6.2

Table 2. Train parameters

Locomotive model	GT26CW
Locomotive weight (ton)	119
Number of locomotive axels	6
Number of wagons	12
Weight of each wagon (ton)	48
Number of wagon axels	4
Basic resistance (N/ton)	$R=2.148+0.002.V+0.0002.V^2$

Table 3. Tuned parameters of GA-PSO

Population size	w	c ₁	c ₂	Iteration
50	0.72	1.5	1.5	30

Figure 8 describes the cost result of applying GA-PSO. The cost value is decreasing from the first iteration and soon keeps stable at the eighth iteration, which reflects a good convergence.

In Figure 9, speed profile generated by GA algorithm in 6.5 minutes is shown. The results are presented in Table 5. This Figure shows that the result is similar to GA-PSO speed profile, but in GA, more notch 8 is used, and more fuel is consumed.

Table 5. GA simulation results

Total energy consumption (MJ)	76.18
Fuel consumption (liter)	10.24
Trip Time (min)	6.5
Distance (km)	6.2

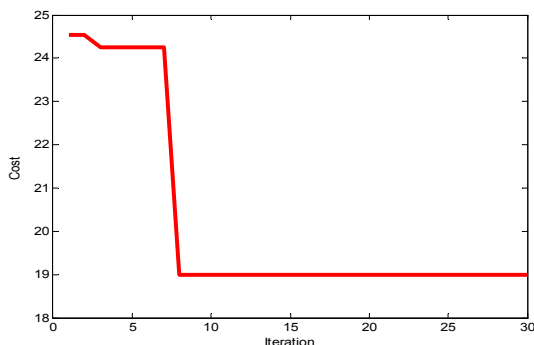


Figure 8. Cost value of GA-PSO Algorithm

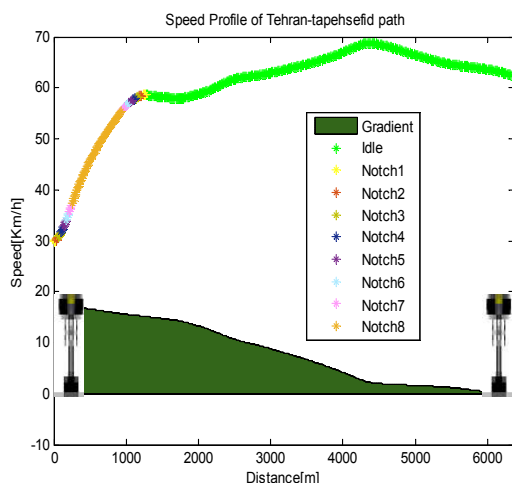


Figure 9. GA optimal speed profile

7. Validation of GA-PSO Algorithm

In order to validate the developed model, Tehran-Tappe_sefid block is selected. All information about control settings of the driver along the journey, speed, position and amount of train fuel consumption obtained through installed sensors on locomotive. As mentioned before, notch is a variable that GA-PSO should determine to generate speed profile. In this section, in order to validate GA-PSO algorithm, the control settings (notches) is applied to the GA-PSO algorithm. If the output algorithm speed profile from the algorithm follows is similar to the actual train driver profile, it can be concluded that the model is accurate. All modellings that have been done accurate and algorithm works well, otherwise it does not work properly. As shown in Figure 8, GA-PSO speed profile tracks the real driver speed profile.

As shown in Figure 10, GA-PSO algorithm (blue continues line) tracks the driver speed profile. It should be noted that every color on driver speed profile belongs to a specific notch (e.g. the color of coasting is shown as green).

8. Conclusions

This paper proposes an energy-efficient approach to reduce the traction energy for Tehran-Tappe_sefid block. In order to have accurate practical speed profiles, Basic resistance, curve resistance force and tractive force is modeled for GT26CW.

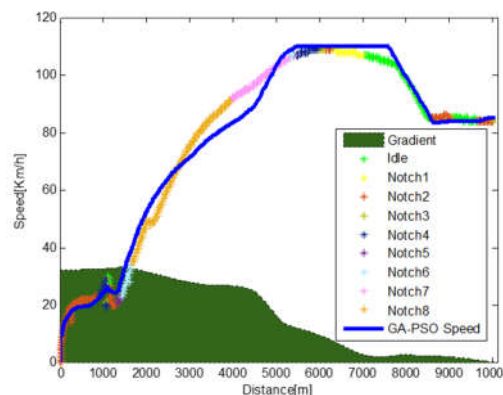


Figure 10. Validation of GA-PSO algorithm

In order to validate models, control settings of driver for a real trip is applied to GA-PSO algorithm and that GA-PSO speed profile tracked the drive speed profile that shows the GA-PSO worked properly. Finally, GA-PSO speed profile compared to GA speed profile. A comparison among real Tehran-Tappe_sefid driver speed profile, GA and GA-PSO (Figures 7,9,10) shows, the profiles of optimized algorithms have been longer in coasting mode than the driver's speed profile, As a result, less energy consumed by observing the speed limit. Total driver fuel consumption for Tehran-Tappe_sefid block is 19.84 liters. Table 6 shows an overview of fuel consumption and time interval over Tehran-Tappe_sefid between block and distance signals for two optimal algorithms in comparison with driver fuel consumption. The results shows that GA-PSO optimization algorithm has better energy-efficient performance. With an increase of 9.2% over the travel time, a 19% and 11% savings in fuel consumption was achieved, for the GA-PSO and GA algorithms, respectively.

This should be noted that one of the reasons for increasing the driver fuel consumption is fatigue of the railroad fleet and the way of driving.

Table 6. Comparison in time and fuel consumption for GA, GA-PSO and driver speed profiles

Driver	GA	GA-PSO	
5.9	6.5	6.5	Time (min)
11.8	10.24	9.50	Fuel Consumption (liter)

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