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Constrained Controller Design for Real-time Delay Recovery in Metro Systems

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A B S T R A C T

This study is concerned with the real-time delay recovery problem in metro loop lines. Metro is the backbone of public transportation system in large cities. A discrete event model for traffic system of metro loop lines is derived and presented. Two effective automatic controllers, linear quadratic regulator (LQR) and model predictive controller (MPC), are used to recover train delays. A newly-designed real-time algorithms for constrained LQR and constrained MPC in the presence of operational constraint on control actions is introduced. MPC and LQR automatic controllers are compared for traffic regulation in metro loop lines. To facilitate fair comparison of the performance of the automatic controllers, different scenarios are considered. The results are then compared to determine the relative effectiveness of the controllers.

1. Introduction

Railway transportation is one of the most important components of public transportation, especially in the form of metros in large cities. Metro lines are inherently unstable. This means that any deviation from the nominal schedule of a given train is amplified over time and disturbs the operation of the other trains [1]. Unwanted disturbance causes deviations from the timetable, propagates delays and increases passenger dissatisfaction; thus, real-time control is necessary to regulate the traffic and prevent instability along the line. Regulation strategies try to recover train delays by varying the nominal running time and/or nominal dwell times at platforms [2].

Railway operations are planned at four levels of hierarchy: strategic, tactical, operational control and real-time control. Strategic operations are handled by railway policy-makers and relate to long-term decisions, whereas tactical operations are developed annually, monthly, or weekly as applicable. Operational control and real-time control regulate the traffic to maintain the nominal timetable in the presence of disturbances and any deviation from the existing timetable. Researchers consider scheduling to be a strategic or tactical operation and rescheduling to be an operation requiring operational or real-time control [3]. The complexity of the metro traffic system demands rescheduling and regulation approaches that can find optimal solution for delay recovery problem. Traffic regulation in a metro line is the task of restoring feasibility in the face of disturbance and limiting the propagation of delays throughout a traffic network [4].

Automatic control methodologies are currently used to regulate the metro traffic. In metro-type railways operated according to a published timetable, complete timetable recovery is the main aim of regulation [2]. These methods are required to minimize the number of delayed passengers, decrease the sum of all train delays and decrease total delay cost [5].

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Van Breusegem et al. [1] undertook metro traffic modeling and regulation for a high-frequency metro traffic system with open lines and loop lines. They designed optimal state feedback which guarantees the stability of the traffic system. Fernandez et al. [2] introduced predictive traffic regulation for metro loop lines. They used quadratic programming to optimize the cost function along a time horizon in the presence of operational constraints. The operational constraints were bounds on control actions, minimum intervals and constraints related to signaling systems. Grube and Cipriano [6] introduced two new strategies, heuristic rules and Model Predictive Control (MPC), for real-time traffic regulation in a metro system. These strategies used holding times of trains at stations to minimize passenger wait times. It was shown that MPC strategy is more effective than the heuristic strategy by reducing the waiting times and travelling times.

Assis & Milani [7] introduced a linear-programming-based model predictive control for optimal train schedules in metro lines. This methodology effectively generates metro line train schedules for a whole day of operation or for one major segment. It can modify the train schedule online during commercial operation for a segment of a given schedule in response to unexpected local passenger demand disturbances.

Lin & Sheu [8] proposed an adaptive optimal control (AOC) algorithm for traffic regulation. This algorithm learned traffic data using artificial neural networks and could approximate the optimal traffic regulator. The efficiency of the AOC algorithm for traffic regulation was verified in a simulated system using traffic data acquired from a real metro line. Sheu & Lin [9] developed an AOC method to design automatic train regulation (ATR) and find a near-optimal solution more rapidly and accurately than does dual heuristic programming. Van den Boom & De Schutter [10] discussed a model of predictive control (MPC) for dynamic traffic management of railway networks. The goal of this methodology is to delay recovery by changing the departure time of trains and breaking the connections.

The linear quadratic regulator (LQR) and MPC are multivariable controllers that have been effectively used in many industrial applications. The key difference between the MPC and LQR is that predictive control solves the optimization problem using a moving time horizon window and LQR solves the same problem within a fixed window. The advantage of the MPC is its ability to perform real-time optimization with hard constraints on plant variables. The LQR is suitable for problems with limited computational power [11].

The present study designs automatic traffic regulator based on the LQR and MPC to recover delays in the loop line of a metro system by changing running times in the presence of operational constraint for control effort. The effectiveness of the controllers in the presence of disturbances and operational constraints is designed and compared.

2. Traffic Model

This section introduces a dynamic model for a traffic loop line relating to the departure times of each train from different platforms. The model is a closed line with N platforms, where platform $\{N\}$ is connected to platform $\{1\}$, and where a given set of trains (indices $\{1\}$ to $\{M\}$) operates periodically [1].

Traffic modeling comprises three main parts of departure time, dwell time and running time. Departure time can be expressed as shown in Equation(1). All upper indices denote train number and lower indices denote platform number.

$$t_{k+1}^{i} = t_{k}^{i} + r_{k}^{i} + d_{k+1}^{i} \tag{1}$$

where:

- t_k^i denotes departure time of train $\{i\}$ at platform $\{k\}$.
- r_k^i is running time of train $\{i\}$ between platform $\{k\}$ and $\{k+1\}$.
- d_{k+1}^{i} is dwell time of train $\{i\}$ at platform $\{k+1\}$.

It is assumed that the number of trains, headway and number of passengers arriving at a given platform per second are constant and that the running time of a train between two successive platforms does not depend on the number of passengers on the train [1].

The dwell time of train $\{i\}$ at platform $\{k+1\}$ can be expressed as:

$$d_{k+1}^{i} = D + c_{k+1}\left(t_{k+1}^{i} - t_{k+1}^{i-1}\right) + w_{k+1}^{j} \tag{2}$$

where the dwell time depends on the nominal dwell time at each platform as denoted by D; c_{k+1} is delay rate and represents the effect of the time interval between two successive trains $\{i-1\}$ and $\{i\}$ at platform $\{k+1\}$ on the dwell time; and w_{k+1}^{i} is the disturbance term.

In Equation (3), r_k^i is the running time of train $\{i\}$ and is composed of nominal running time R_k between platform $\{k\}$ and $\{k+1\}$ and control signal u_k^i applied to the traffic system to increase or decrease running time. As shown, $u_k^i > 0$ increases the running time and, conversely, $u_k^i < 0$ decreases the running time.

$$r_k^i = R_k + u_k^i \tag{3}$$

Equations (1)-(3) can be used to obtain Equation (4) as:

$$t_{k+1}^{i} = \frac{t_k^i}{1-c_{k+1}} - \frac{c_{k+1}t_{k+1}^{i-1}}{1-c_{k+1}} + \frac{(D+R_k)}{1-c_{k+1}} + \frac{u_k^i}{1-c_k} \\ \frac{w_k^i}{1-c_{k+1}} \tag{4}$$

The nominal headway in each platform is expressed in Equation(5), where $t_{k,n}^{i}$ is the nominal departure time for train $\{i\}$ at platform $\{k\}$:

$$H = t_{k,n}^{i+1} - t_{k,n}^{i} \tag{5}$$

Time deviation (TD) Δt_k^i is defined as the difference between the actual departure time and the nominal predefined departure time as:

$$\Delta t_k^i = t_k^i - t_{k,n}^i \tag{6}$$

Using Equations (1), (2) and (5), Equation (6) can be obtained to show the relation between the nominal departure time for train $\{i\}$ at two successive platforms $\{k\}$ and $\{k+1\}$ as:

$$t_{k+1,n}^{i} = t_{k,n}^{i} + c_{k+1}H + D + R_k \tag{6}$$

From Equations (1)-(6), Equation (7) can be obtained to represent the dynamic behavior of TD as:

$$\Delta t_{k+1}^{i} = -\frac{c_{k+1}}{1-c_{k+1}}\Delta t_{k+1}^{i-1} + \frac{1}{1-c_{k+1}}\Delta t_k^i + \frac{1}{1-c_{k+}} \\ + \frac{1}{1-c_{k+1}} w_k^i \quad , \quad 0 \le k \le N \ , \ 1 \le i \le \tag{7}$$

For better understanding of the relation between the variables and to represent a state space model for TDs in a loop line without loss of generality, consider a special case containing 2 trains $\{M=2\}$ and 10 platforms $\{N=10\}$ as:

$$\begin{aligned}
i = 1 \quad & k=1 \ : \ \Delta t_2^1 = \kappa\,\Delta t_1^1 + \lambda\Delta t_2^0 + \kappa u_1^1 + \kappa w_1^1 \\
& k=2 \ : \ \Delta t_3^1 = \kappa\,\Delta t_2^1 + \lambda\Delta t_3^0 + \kappa u_2^1 + \kappa w_2^1 \\
& \vdots \\
& k=10 \ : \ \Delta t_{11}^1 = \kappa\,\Delta t_{10}^1 + \lambda\Delta t_{11}^0 + \kappa u_{10}^1 + \kappa w_{10}^1 \\
i = 2 \quad & k=0 \ : \ \Delta t_1^2 = \lambda\Delta t_1^1 \\
& k=1 \ : \ \Delta t_2^2 = \kappa\,\Delta t_1^2 + \lambda\Delta t_2^1 + \kappa u_1^2 + \kappa w_1^2 \\
& k=2 \ : \ \Delta t_3^2 = \kappa\,\Delta t_2^2 + \lambda\Delta t_3^1 + \kappa u_2^2 + \kappa w_2^2 \\
& \vdots \\
& k=10 \ : \ \Delta t_{11}^2 = \kappa\,\Delta t_{10}^2 + \lambda\Delta t_{11}^1 + \kappa u_{10}^2 + \kappa w_{10}^2 \\
& \textit{where} \\
& \qquad \kappa = \frac{1}{1-c_{k+1}}, \ \lambda = -\frac{c_{k+1}}{1-c_{k+1}}
\end{aligned} \tag{8}$$

By defining state vector $\boldsymbol{X}_j^i$, input vector $\boldsymbol{U}_j^i$ and disturbance vector $\boldsymbol{W}_j^i$ as:

$$\begin{aligned}
\boldsymbol{X}_j^i &= \left[\Delta t_{j-N}^i \quad \Delta t_{j-N+1}^i \quad \ldots. \quad \Delta t_{j-M-1}^i \mid \right. \\
& \left. \Delta t_{j-M}^i \quad \ldots \quad \Delta t_{j-1}^{i-M+1}\right]^T \\
\boldsymbol{U}_j^i &= \left[u_{j-M}^i \quad u_{j-M+1}^{i-1} \quad \ldots \quad u_{j-1}^{i-M+1}\right]^T \\
\boldsymbol{W}_j^i &= \left[w_{j-M}^i \quad w_{j-M+1}^{i-1} \quad \ldots \quad w_{j-1}^{i-M+1}\right]^T
\end{aligned} \tag{9}$$

the corresponding state space model becomes:

$$\boldsymbol{X}_{j+1}^i = A_L \boldsymbol{X}_j^i + B_L[\boldsymbol{U}_j^i + \boldsymbol{W}_j^i] \quad i \in [1, M] \tag{10}$$

where:

$$A_L = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]$$

$$A_{11} = \begin{bmatrix} 0 & 1 & & \\ \vdots & \ddots & \ddots & \\ & & 0 & 1 \\ 0 & & \cdots & 0 \end{bmatrix}_{R\times R} \quad A_{12} = \begin{bmatrix} 0 & \cdots & & 0 \\ \vdots & \ddots & & \\ 0 & & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}_{R\times M}$$

$$A_{21} = \begin{bmatrix} 0 & \cdots & & 0 \\ \vdots & \ddots & & \\ 0 & & \ddots & \vdots \\ \lambda & 0 & \cdots & 0 \end{bmatrix}_{M\times R} \quad A_{22} = \begin{bmatrix} \kappa & \lambda & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \lambda \\ 0 & \cdots & 0 & \kappa \end{bmatrix}_{M\times M} \tag{11}$$

$$B_L = \left[\begin{array}{c} B_1 \\ \hline B_2 \end{array}\right]$$

$$B_1 = \begin{bmatrix} & & \\ & 0 & \\ & & \end{bmatrix}_{R\times M} \quad B_2 = \begin{bmatrix} \kappa & & 0 \\ & \ddots & \\ 0 & & \kappa \end{bmatrix}_{M\times M}$$

$$(R = N - M)$$

3. Objective Function and LQR Design

The main objectives of any traffic regulator in a metro system are to minimize the TDs and increase passenger satisfaction with minimum control effort. Consequently, the cost function is defined as shown in Equation(12). It includes three quadratic terms subject to TD, passenger satisfaction to balance deviation at all platforms in relation to one another, and control action.

Cost function (12) is similar to the function defined by [1] and [2]. Fernandez et al .[2] explained that, if the line is operating according to a predefined timetable, the value of weighting matrix P must be large enough to allow total delay compensation when compared with the value of the other weighting matrices. If the line is operating according to a predefined headway, weighting matrix P could theoretically be equal to zero and weighting matrix Q must be sufficiently large. In this case, the solution reaches headway regularity, but does not completely compensate for the delays as:

$$\begin{aligned} J &= (\boldsymbol{X}_{j+1}^i)^T P(\boldsymbol{X}_{j+1}^i) + (\boldsymbol{X}_{j+1}^i - S_N \boldsymbol{X}_j^i)^T Q \\ &(\boldsymbol{X}_{j+1}^i - S_N \boldsymbol{X}_j^i) + (\boldsymbol{U}_j^i)^T \boldsymbol{U}_j^i \\ \Rightarrow J &= \left(A_L \boldsymbol{X}_j^i + B_L \boldsymbol{U}_j^i\right)^T P \left(A_L \boldsymbol{X}_j^i + B_L \boldsymbol{U}_j^i\right) \\ &\left(A_L \boldsymbol{X}_j^i + B_L \boldsymbol{U}_j^i - S_N \boldsymbol{X}_j^i\right)^T Q \\ &\left(A_L \boldsymbol{X}_j^i + B_L \boldsymbol{U}_j^i - S_N \boldsymbol{X}_j^i\right) + (\boldsymbol{U}_j^i)^T \boldsymbol{U}_j^i \end{aligned} \tag{12}$$

where:

$$S_N = \begin{bmatrix} 0 & 1 & .. & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \vdots & 0 & 1 \\ 1 & 0 & .. & 0 \end{bmatrix}_{N\times N} z$$

$$P = \left[\begin{array}{c|c} 0_{R\times R} & 0 \\ \hline 0 & p \times I_{M\times M} \end{array}\right]_{N\times N} \quad Q = \left[\begin{array}{c|c} 0_{R\times R} & 0 \\ \hline 0 & q \times I_{M\times M} \end{array}\right] \tag{13}$$

3.1. LQR without constraints

The optimal state feedback based on LQR control methodology without consideration of constraints on control action and state variables is obtained by minimizing objective function (12) [1] as:

$$\begin{aligned} \boldsymbol{U}_j^i = -\left[I_M + B_L^T (P+Q) B_L\right]^{-1} . \\ \left[B_L^T (P+Q) A_L - B_L^T Q S_N\right] \end{aligned} \tag{14}$$

3.2. Constrained LQR

Metro traffic regulation, as for nearly all real world optimal control systems, has constraints which should be satisfied. In this study, bounds on the amplitude of control actions have been considered. Constraints on control actions can be expressed as $U_{\min} \le \boldsymbol{U}_j^i \le U_{\max}$, where $U_{\min}$ and $U_{\max}$ are vectors in which the elements are the lower bounds and upper bounds of control actions, respectively.

In constrained optimization, cost functions are solved in the presence of constraints. There are different methods for solving these kinds of problems. Sequential quadratic programing (SQP) with quadratic objective functions is an effectiveness algorithm. The aim of this method is to solve a nonlinearly constrained problem using a sequence of quadratic programming (QP) [12]. The following algorithm details the steps of constrained LQR design using SQP methodology.

Algorithm of constrained LQR

Step 0: Set $i = M$ and $j = 1$

Step 1: Initialize $\boldsymbol{X}_j^i = 0$

Step 2: Define the value of disturbances that have occurred as (W_j^i)

Step 3: Minimize cost function *J*, (12), based on the SQP algorithm and obtain U_j^i

Step 4: Insert X_j^i, W_j^i and U_j^i into (10) and obtain X_{j+1}^i

Step 5: Set $j = j+1$ and return to step 2 until $j = L \times N$, where L is number of the loops which trains orbit

4. MPC

MPC is a popular controller design method in the process industry. Predictive control incorporates the prediction of system behavior into its formulation. The estimate of future system variables can then be used in the design of control laws to achieve good control performance, which is usually to drive or maintain the output to a desired set point [13]. An important advantage of MPC is that the use of a finite horizon allows the inclusion of additional constraints [14].

A common class of model predictive control is generalized predictive control (GPC). The GPC recursive formulation can be explained as follows.

Consider a state space model of a system as:

$$\begin{aligned} x(j+1) &= Ax(j) + Bu(j) \\ y(j) &= Cx(j) \end{aligned} \tag{15}$$

The structure of model (15) is used to formulate the predictive controller. The definition of a state prediction model is:

$$\begin{aligned} \hat{x}(j+r \mid j) = A\hat{x}(j+r-1 \mid j) + \\ Bu(j+r-1 \mid j) \quad r = 1,2,.. \end{aligned} \tag{16}$$

where $\hat{x}(j+r \mid j)$ denotes state vector prediction at instant j for instant $j+r$. The prediction of output is shown in Equation (17) as the recursive operation:

$$\begin{aligned} &\hat{y}(j+1 \mid j) = CAx(j) + CBu(j) \\ &\hat{y}(j+2 \mid j) = CA^2x(j) + \\ &\qquad CABu(j) + CBu(j+1) \\ &\vdots \\ &\hat{y}(j+p_1 \mid j) = CA^{p_1}x(j) + CA^{p_1-1}Bu(j) \\ &\qquad .....+ CA^{p_1-p_2-1}Bu(j+p_2 \end{aligned} \tag{17}$$

where p_1 is the output prediction horizon and p_2 is the control prediction horizon. The matrix form of (17) is:

$$Y = Gx(j) + FU \tag{18}$$

where:

$$Y = [\hat{y}(j+1 \mid j) \quad \hat{y}(j+2 \mid j) \quad ..... \quad \hat{y}(j+p_1 \mid j)]^T$$

$$U = [u(j) \quad u(j+1) \quad ... \quad u(j+p_2)]^T$$

$$G = \begin{bmatrix} CA \\ CA^2 \\ : \\ CA^{p_1} \end{bmatrix}$$

$$F = \begin{bmatrix} CB & 0 & .. & 0 \\ CAB & CB & .. & 0 \\ : & : & ... & : \\ CA^{p_1-1}B & CA^{p_1-2}B & .. & CA^{p_1-p_2-1}B \end{bmatrix}$$

Predictive control law can be obtained by minimizing the appropriate objective function. In this case, the first *M* rows (for multivariable system with *M* input variables) of matrix U are applied to the system. In the next step, the control effort should again be calculated [13].

4.1. MPC without constraints

Section 2 presented the state space model of metro traffic as depicted in Equation(10). A comparison of state space models (10) and (15) produces the following for design of the GPC for traffic regulation:

$$\begin{aligned} A &= A_L \\ B &= B_L \\ C &= I \end{aligned} \tag{19}$$

The traffic model is a linear multivariable system with M input variables and N output variables. The compact formulation of the output prediction subject in Equation (18) for the traffic system is:

$$Y_j^i = GX_j^i + F\bar{U}_j^i \tag{20}$$

where:

$$\bar{U}_j^i = \begin{bmatrix} \boldsymbol{U}_j^i & \boldsymbol{U}_{j+1}^i & \cdots & \boldsymbol{U}_{j+p_2}^i \end{bmatrix}^T$$

$$\begin{aligned} Y_j^i &= [\hat{y}_{j+1|j}^i \quad \hat{y}_{j+2|j}^i \quad ..... \quad \hat{y}_{j+p_1|j}^i]^T \\ &= [\hat{\boldsymbol{X}}_{j+1|j}^i \quad \hat{\boldsymbol{X}}_{j+2|j}^i \quad ..... \quad \hat{\boldsymbol{X}}_{j+p_1|j}^i]^T \end{aligned} \tag{21}$$

As in LQR design procedure, the objective function of traffic regulation is as shown in Equation(12); however, for GPC design, it should be rewritten in predictive form as:

$$\begin{aligned} J &= (Y_j^i)^T \bar{P}(Y_j^i) + \\ &(Y_j^i - \bar{S}_N Y_{j-1}^i)^T \bar{Q}(Y_j^i - \bar{S}_N Y_{j-1}^i) + (\bar{U}_j^i)^T (\bar{U} \end{aligned} \tag{22}$$

where $\bar{P}, \bar{Q}$ and $\bar{S}_N$ are:

$$\bar{P} = \left[\begin{array}{c|c|c|c} P & 0 & \cdots & 0 \\ \hline 0 & P & \ddots & 0 \\ \hline 0 & \ddots & \ddots & 0 \\ \hline 0 & \cdots & 0 & P \end{array}\right]_{(p_1 \times N)\times(p_1 \times N)}$$

$$\bar{Q} = \left[\begin{array}{c|c|c|c} Q & 0 & \cdots & 0 \\ \hline 0 & Q & \ddots & 0 \\ \hline 0 & \ddots & \ddots & 0 \\ \hline 0 & \cdots & 0 & Q \end{array}\right]_{(p_1 \times N)\times(p_1 \times N)} \tag{23}$$

$$\bar{S}_N = \left[\begin{array}{c|c|c|c} S_N & 0 & \cdots & 0 \\ \hline 0 & S_N & \ddots & 0 \\ \hline 0 & \ddots & \ddots & 0 \\ \hline 0 & \cdots & 0 & S_N \end{array}\right]_{(p_1 \times N)\times(p_1 \times N)}$$

The control law is obtained as shown in Equation (24) by minimizing cost function (22). Notice that only the first M rows of $\bar{U}_j^i$ are applied to the metro traffic system; the other elements should be removed. In each iteration, $\bar{U}_j^i$ should again be calculated:

$$\frac{\partial J}{\partial \bar{U}_j^i} = 0$$

$$\begin{aligned} &\Rightarrow 2F^T\bar{P}GX_j^i + 2F^T\bar{Q}GX_j^i - 2F^T\bar{Q}\bar{S}_N Y_j \\ &\quad + 2F^T\bar{P}F\bar{U}_j^i + 2F^T\bar{Q}F\bar{U}_j^i + 2\bar{U}_j^i = 0 \end{aligned} \tag{24}$$

$$\begin{aligned} \Rightarrow \bar{U}_j^i &= -[\mathrm{I} + F^T(\bar{P}+\bar{Q})F]^{-1}. \\ &[F^T(\bar{P}+\bar{Q})GX_j^i - F^T\bar{Q}\bar{S}_N Y_{j-}^i \end{aligned}$$

4.2. Constrained MPC

As in constrained LQR design procedure, the bounds on the amplitude of control signals are considered to be $U_{\min} \le \boldsymbol{U}_j^i \le U_{\max}$, where $U_{\min}$ and $U_{\max}$ are vectors with elements for the lower bounds and upper bounds of control action, respectively. The SQP algorithm is used to minimize the objective function [15]. The following algorithm shows the details of the steps for constrained MPC design using SQP.

Algorithm of constrained MPC

Step 0: Set $i = M$ and $j = 1$

Step 1: Initialize $\boldsymbol{X}_j^i = 0$

Step 2: Define the value of disturbances that have occurred as ($\boldsymbol{W}_j^i$)

Step 3: Enter $\boldsymbol{X}_j^i$ into Equation (20) to obtain Y_j^i

Step 4: Minimize the cost function *J*, (22), based on the SQP algorithm to obtain $\bar{U}_j^i$

Step 5: Apply only M rows of $\bar{U}_j^i$ to the system (U_j^i = first M rows of $\bar{U}_j^i$)

Step 6: Insert $\boldsymbol{X}_j^i$, $\boldsymbol{W}_j^i$ and $\boldsymbol{U}_j^i$ into Equation (10) to obtain $\boldsymbol{X}_{j+1}^i$

Step 7: Set $j = j + 1$ and return to step 2 until $j = L \times N$, where L is the number of the loop around which trains orbit

5. Simulation Results

The simulations consider a traffic loop line in which the number of platforms is $N = 10$ and the number of trains is $M = 4$. The parameters of the traffic model and objective function are $c_{k+1} = 0.1$, $N = 10$, $M = 4$, $p = 10$, $q = 4$, $L = 4$ and $N \times L = 40$. Simulation results are presented for two main cases. Figure 1 shows the constraints on control actions.

Case 1: LQR and constrained LQR

Figure 2 shows the time deviations for four trains when LQR and constrained LQR are applied to the system. Figure 3 shows the control actions for these two controllers. The amount of delay is now increased in the traffic system as $w_7^1 = 100$, $w_4^2 = 150$, $w_3^3 = 150$ and $w_5^4 = 300$. The simulation result is shown in Figure 4. It is clear that the constrained LQR was unable to recover from the delays.

Case 2: MPC and constrained MPC

The MPC and constrained MPC are used to tackle large disturbances $w_7^1 = 100$, $w_4^2 = 150$, $w_3^3 = 150$ and $w_5^4 = 300$. Figures 5 and 6 show the simulation results in which the MPC and constrained MPC are able to dampen the time delays and regulate the metro traffic that the constrained LQR cannot regulate. Table 1 compares the LQR and MPC controllers. Note that the constrained LQR could not handle large time delays. All simulations were conducted on a PC (CPU: Intel Core i7; 2.00 GHz; RAM: 6 GB; 64-bit Windows 8 OS) in the MATLAB 8.0.1.604 software environment.

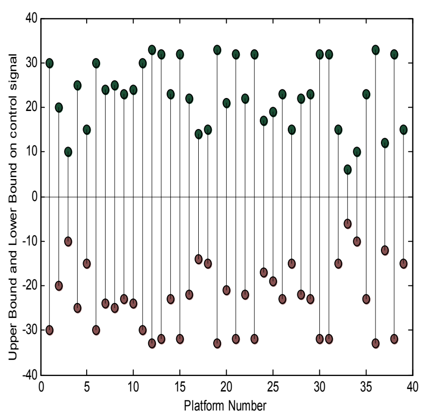

Figure 1. Upper and lower bounds of control signals

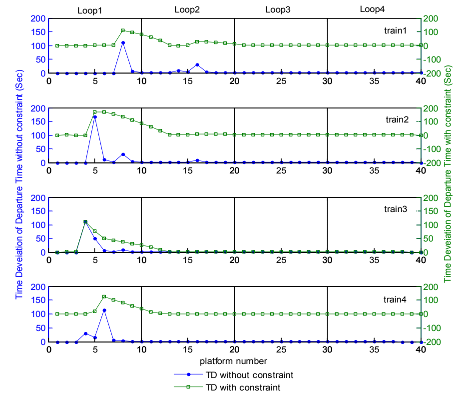

Figure 2. Time deviation for departure times in LQR and constrained LQR.

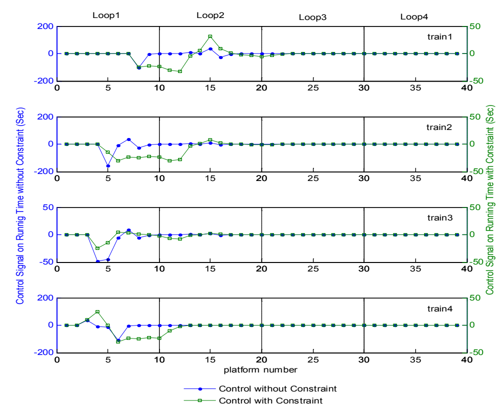

Figure 3. Control signals for LQR and constrained LQR

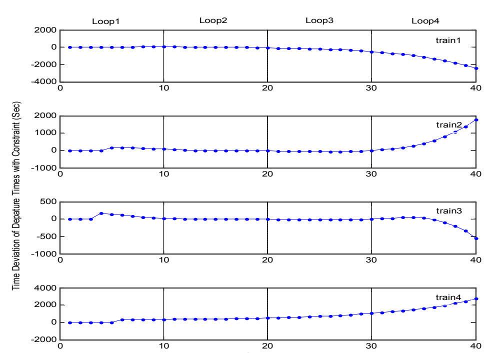

Figure 4. Time deviation of departure times for constrained LQR.

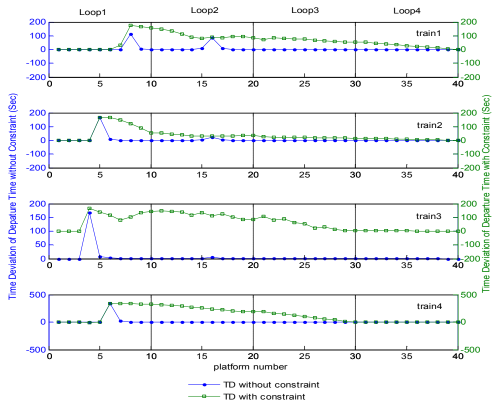

Figure 5. Time deviation of departure times for MPC and constrained MPC.

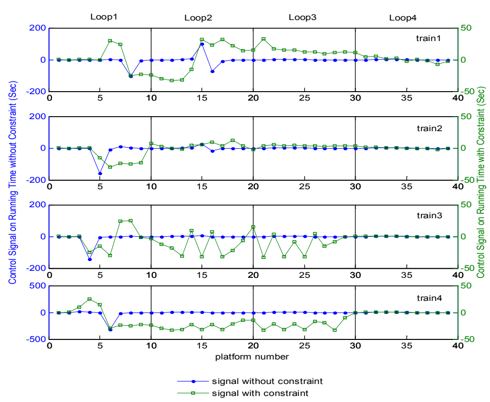

Figure 6. Control signal for MPC and constrained MPC.

6. Conclusions

This study focused on traffic regulation of metro systems. A real-time delay recovery problem for metro loop lines using constrained Model Predictive Controller (MPC) and constrained Linear Quadratic Regulator (LQR) controllers in the presence of operational constraints on control action is considered. These two types of controllers were implemented to metro traffic model for recovering delays. The performance of the MPC and LQR were compared for traffic regulation. Simulation results show that constrained MPC performed better in comparison with the constrained LQR. Whereas, the MPC was computationally intensive, especially for large-scale metro traffic systems.

Table 1. Comparison of LQR and MPC controllers

Controller	Type of controller	Performance	Elapsed computation time (Sec)	Computing Software
LQR	Without any Constraint	Tackle any time delays	0.005980	MATLAB
	With Constraint	Tackle the normal values of time delays	2.201511	
		Failed in deal with large values of time delays	---	
MPC	Without any Constraint	Tackle any time delays	0.059	
	With Constraint	Tackle the large values of time delays	6.521701	

References

[1] V. Van Breusegem, G. Campion, G. Bastin, Traffic modeling and state feedback control for metro lines, IEEE T. Automat. Contr., Vol.36, No.7, (1991), pp. 770-784.

[2] A. Fernandez, A.P. Cucala, B. Vitoriano, F. De Cuadra, Predictive traffic regulation for metro loop lines based on quadratic programming, P. I. MECH. ENG. F-J. RAI, Vol.220, No.2, (2006), pp. 79-89.

[3] S. Narayanaswami, N. Rangaraj, Scheduling and rescheduling of railway operations: a review and expository analysis, Technol. Oper. Manag., Vol.2, No.2, (2011), pp. 102-122.

[4] F. Corman, R.M. Goverde, and A. D'Ariano, Rescheduling dense train traffic over complex station interlocking areas , in Möhring, R. K. A. R. H., Zaroliagis, C. D. (Eds), 'Robust and Online Large-Scale Optimization', Springer Berlin Heidelberg, (2009), pp. 369-386.

[5] B. Fanm, Railway Traffic Rescheduling Approaches to Minimize Delays Disturbances Conditions, PhD thesis, University of Birmingham, (2012).

[6] P. Grube, A.M. Cipriano, Comparison of simple and model predictive control strategies for the holding problem in a metro train system, IET Intell. Transp. Sy., Vol.4, No.2, (2010), pp. 161-175.

[7] W.O. Assis, B.E. Milani, Generation of optimal schedules for metro lines using model predictive control, Automatica, Vol. 40, No.8, (2004), pp. 1397-1404.

[8] W.S. Lin, J.M. Sheu, Metro traffic regulation by adaptive optimal control, IEEE T. Intell. Transp., Vol.12, No.4, (2011), pp. 1064-1073.

[9] W.S. Lin, J.W. Sheu, Adaptive optimal control for designing automatic train regulation for metro line, IEEE T. Contr. Syst. T., Vol.20, No.5, (2012), pp.1319-1327.

[10] T. Van den Boom, B. De Schutter, On a model predictive control algorithm for dynamic railway network management, In 2nd International Seminar on Railway Operations Modelling and Analysis (Rail-Hannover), Hannover, Germany, (2007), pp. 15.

[11] L. Wang, Model predictive control system design and implementation using MATLAB, Springer Science & Business Media, London, (2009).

[12] P.E. Gill, W.A. Murray, M. Saunders, SNOPT: An SQP algorithm for large-scale constrained optimization, SIAM Rev., Vol.47, No.1, (2005), pp. 99-131.

[13] S. Huang, K.K. Tan, T.H. Lee, Applied predictive control, Springer Science & Business Media, London, 1st Edi, (2002).

[14] B. De Schutter, T. Van Den Boom, Model predictive control for max-plus-linear discrete event systems, Automatica, Vol.37, No.7, (2001), pp. 1049-1056.

[15] J. Nocedal, S.J. Wright, Numerical Optimization, Springer Series in Operations Research, New York, 1999, 2nd Edi. (2006).