International Journal of Railway Research, Vol. 8, No. 1, (2021), 53-64

Iran University of Science and Technology

International Journal of

Railway Research

ISSN: 2423-3838

Simultaneous Schedule of Trains and Track Maintenance according to Stochastic Blockage Time

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ARTICLE INFO

Article history:
Received: 26.02.2021
Accepted: 07.12.2021
Published: 11.12.2021

Keywords:
Track maintenance
Simultaneous scheduling
Stochastic programming
Blockage

ABSTRACT

In the area of the railway infrastructure, planning for track maintenance is a challenging problem due to the coordination between train traffic and maintenance operations. In this paper, we discuss the simultaneous scheduling of trains and operations for a single-track line, where stochastic mathematical programming is applied to minimize both the travel time of trains and maintenance duration. The uncertainty in the blockage duration of track is considered in the model to reduce the impact of unexpected delays in maintenance operations. The proposed model also considers the practical aspects which has been less addressed in the previous researches regarding track maintenance. We reformulated the stochastic problem to the deterministic equivalent to facilitate solutions for realistic sizes. The computational result for a real track shows that the model can efficiently find simultaneous scheduling of trains and operations and suggest optimal blockage duration at different confidence levels. The impact of timetable comparison on the blockage duration for a heavily utilized line is also evaluated.

1. Introduction

Regular maintenance of rail infrastructure ensures the appropriate operation of the rail system, safety, and satisfaction of passengers. The track is one of the most maintenance-intensive parts of the railway system, which is expected to preserve or even improve the performance and capacity of rail infrastructure. A key issue in track maintenance is the process of detecting defects, determining the required maintenance types, and then scheduling the operation. This process might vary according to the rules and regulations of maintenance management. For example, in the Railway of Iran (RAI), defects are firstly detected in four ways by the Department of Track Maintenance depending on the availability of resources: (i) monitoring the track with measuring machines, (ii) inspection of the track by maintenance experts, (iii) monthly inspection of the track by a team from a high-ranking commission called the Commission of Accidents, and (iv) reports from train drivers.

In the next step, the type of maintenance and the minimum required time of the operation for each track segment is announced to Dispatching Department to adjust the timetable according to the required maintenance operation. It should be noted that during the maintenance operation, the track would be out of service as it is often carried out by heavy rail machinery. Accordingly, the initial timetable of trains should be adjusted to include maintenance activities. The modified timetable is currently prepared based on the experience of experts at the Dispatching

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Department of RAI and is further checked by simulation software.

It is apparent that although the obtained timetable might be feasible, it lacks some main features. First, the duration of track blockage for maintenance is assumed to be a constant value based on the engineering judgment of the expert. In practice, the blockage duration is mainly variable due to some unexpected conditions such as machinery break down, unavailability of crews, inclement weather, etc. If the probable delay in maintenance duration is ignored, it can be propagated throughout the timetable and impose additional costs on the operator. Moreover, scheduling trains and maintenance activities is a critical task as they utilize the capacity of the track competitively. Operating more trains leads to less time available for performing maintenance, and vice versa. The tension is especially high when track capacity is inadequate, which is the case in many bottleneck areas now. Therefore, it is hardly possible to efficiently adjust a timetable for trains and maintenance activities merely based on the experiment of experts. Furthermore, dispatching trains according to their importance is the main issue in the simultaneous scheduling of trains and maintenance operations. The allowable speed of trains on track segments requiring maintenance is different before and after the operation. Important trains such as express passenger trains should arrive at the destination with maximum allowable speed and the least dwell time in stations. Regarding the importance of trains in the integration of two tasks – schedule trains and maintenance operation- make it a more challenging issue.

Concerning the above discussion, developing a mathematical optimization model to design the simultaneous timetable of trains and maintenance operations while regarding practical conditions and uncertainty in blockage duration is vital for rail authority. Optimal simultaneous scheduling could be a step forward to improve the interaction between maintenance and train schedules and further towards efficient use of capacity.

There are numerous studies in the literature that have investigated the subject of the train and track maintenance scheduling independently. For the first subject (optimization of train scheduling), Szpigel [1] presented a mathematical programming model to optimize the train scheduling problem. Since then, numerous models have been proposed to consider different operational situations and utilize innovative solution methods. A review of this problem reveals that there are three major trends in the literature: tactical scheduling, operational scheduling, and rescheduling. To avoid prolixity, interested readers are referred to the review paper of Törnquist [2] for more details of each trend. The latter subject (track maintenance scheduling problem) is also an issue that has been studied by many researchers in recent years. A comprehensive review of this subject can be found in Lidén [3].

Despite the extensive studies on each of the two aforementioned subjects, there are limited works on the problem of simultaneous scheduling of trains and maintenance operations due to its complexity in modeling and solving approach. Simultaneous scheduling of trains and maintenance operations in railways was first considered by Higgins, Ferreira [4] to minimize train delays and costs associated with maintenance operations. They predicted the interference of train services and maintenance operations by including a constant probability coefficient into the model to anticipate the delays of trains and maintenance activities. The proposed combinatorial model could only be implemented on small and medium-size networks, and if the number of stations and trains increases, the time for solving the model would be significant. Moreover, they assumed that the blockage duration is simply composed of some discrete intervals that should be known in advance. Since such an interval is determined by the user and through trial and error, its value will affect the accuracy of the final solution. To overcome the computational burden of the previous model for large size networks, Albrecht, Panton [5] proposed an approach for the scheduling of the train services, which includes blockage duration for maintenance operations. In their study, first, a set of feasible timetables are produced in the absence of track maintenance. Then alternative plans are produced by adjusting the departure and arrival time of trains to minimize delays. In fact, instead of using a mathematical programming model, they utilized the problem space search (PSS) technique for creating the best plan. Furthermore, the blockage duration is simulated by a dummy train with the travel time equal to maintenance operation on the track under maintenance. Despite proposing a novel approach in this area, the study by Albrecht, Panton [5] neglects to address the practical aspects of maintenance operations like the compulsory sequence of some maintenance operations. Also, their study might have been more useful if the possibility of rerouting of trains due to the track blockage was considered. To overcome the latter flaw, Forsgren, Aronsson [6] suggested a model to consider the rerouting of trains during maintenance operations while trying to make the least adjustment and train cancelation in the primary timetable.

The length of blockage duration and its effect on railway traffic has also been investigated in some studies. Lidén and Joborn [7] considered the overhead time composed of preparation time (including the time required for setting up the maintenance operation) and termination time (including the time required for removing maintenance facilities), and suggest some part of this overhead time (such as the transfer of the maintenance crew to the determined place of maintenance and installation of safety signs in the area after the last train passed) to be transferred out of the blockage time to increase the efficiency of train schedules. To improve their previous work, Lidén, Kalinowski [8] considered the constraint on the number of maintenance machines in the scheduling. However, the uncertainty of the blockage duration was yet ignored in the scheduling of the trains. Vansteenwegen, Dewilde [9] added a marginal time to the travel time of passenger trains to address the time required for maintenance activities and claimed the proposed timetable is robust. In their model, the modification of the initial train scheduling because of a maintenance operation is carried out by using rerouting, rescheduling, or cancellation of some trains. However, the uncertainty of the blockage duration is not considered in their study. D’Ariano, Meng [10] addressed such deficiency and considered uncertainty in both track maintenance and travel time of trains. The interference of train services and maintenance operations was considered by developing various scenarios at a microscopic level.

Reviewing the above studies demonstrates that few pieces of research have been conducted about optimizing the simultaneous scheduling of trains and maintenance operations till now. Besides, the existing studies have not considered those aspects of maintenance operations that are applied in practice. This paper aims to address the problem of simultaneous scheduling of trains and maintenance operations and is distinct from the above studies in some ways. First, the uncertainty in the blockage duration of track is included in the proposed model to consider unexpected delays due to machinery failure, unavailability of the maintenance crew, inclement weather, or even wrong estimation of the required time for maintenance operation. If a probable delay in maintenance operation is not considered in the train timetable, cascading delays will occur in the movement of trains. In this study, an approach is proposed which considers uncertainty in the blockage duration by including stochastic buffer time to the time required of each maintenance operation.

Another contribution of this study is that it considers practical aspects of maintenance operations in the proposed model to make it applicable in practice. For example, it is necessary to consider the speed limit of trains before and after the maintenance operation. Also, some maintenance activities should be operated successively to achieve the best performance. The sequence of maintenance operations is another practical aspect being addressed in this study. Considering the starting and finishing time of maintenance operation according to working hours and the minimum blockage duration for each maintenance type are the other operational features that enhanced the applicability of the proposed model.

The next contribution of this study which highlights it from the past ones, is considering a continuous time window for the integrated scheduling of maintenance activities and trains. Previous studies commonly used time slots or discrete time intervals to represent the departure or arrival time of trains and maintenance operations in the timetable [4,7,8,11]. Although discrete time intervals might simplify the scheduling problem, it leads to less accurate results. Moreover, the division of the time window into a small time-interval length leads to very large combinatorial problems of intractable size. In this research, a continuous time pattern has been used in the proposed model to overcome that limitation.

The remainder of this paper is organized as follows. Section 2 describes the problem characteristics for assumptions and operational aspects. Section 3 presents the optimization model of the simultaneous scheduling of trains and the proposed solution approach. The results of implementing the proposed model on a real case study will be discussed in Section 4. Finally, Section 5 finishes the paper through conclusions and directions for further works.

2. Problem statement

The problem studied in this paper can be described as follows. Consider a single-track line that consists of a set of track segments between any two successive stations and a set of trains moving between origin-destination (ODs) according to the daily timetable. As Fig. 1 shows, the trains moving from the west are called west trains, and those moving from the east are called east ones. After the inspection of tracks, given the required maintenance operation of track segments, the minimum blockage duration is realized for each segment. Table 1 indicates the set of maintenance operations regarded in this study. Fig. 2 also shows the equipment of each maintenance operation.

It should be noted that some maintenance activities should be operated successively and ceaselessly. For example, ballast regulation is followed by tamping, where a normal speed is immediately required after the operation. The categories in the first two rows of Table 1 indicate such type of operation.

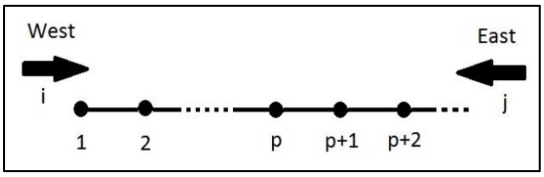


Figure 1: Single rail corridor and track segments.

Table 1: Maintenance categories of the problem under study

Maintenance Category	Description
1	tamping, ballast regulating, track stabilizing
2	tamping, ballast regulating
3	temping
4	ballast regulating
5	track stabilizing
6	Rubbing of rail
7	Welding of rail
8	Rail renewal

Figure 2: Equipment of maintenance operations: (a) ballast cleaner, (b) ballast tamper, (c) track stabilizer, (d) ballast regulator, (e) rail grinder, (f) rail welding machine.

The problem is to seek an optimal simultaneous schedule of trains and maintenance operations for a single-track line such that the travel time of trains and the duration of operations are minimized. In this respect, some assumptions and operational aspects are considered as follows:

1- The priority of trains is given by assigning the important factor to each train. Commonly, passenger trains have priority over freight trains, as delays in passenger trains cost rail authority more than the ones in freight trains.

2- The most allowable travel time of trains at each track segment and the least dwell time at each station are determined according to the operational conditions.

3- The minimum required blockage time for maintenance operations at each track segment is given.

4- Crossing and overtaking of trains are allowed only at stations.

5- The departure and arrival times of trains have a certain threshold. It means that the trains depart the origin station or arrive at the destination within a specific time of day. It is especially the case where passenger satisfaction shall be achieved in luxury services.

6- The allowable speed of trains on track segments requiring maintenance is different before and after the operation.

7- Some of the maintenance operations must be carried out successively to achieve the best quality of operations. For example, ballast regulation should be followed immediately after the temping when the equipment for both operations is available.

8- The starting and finishing time of the maintenance activities are limited to the working hours of the crew.

All the above assumptions are considered as constraints in the problem under study. Recall that each maintenance operation requires the track to be blocked within a certain duration. The blockage duration, however, is subject to changes due to unexpected delays in the maintenance process or operational tasks. If such probable delays are not included in the simultaneous scheduling, cascading delays will occur in the train services. To avoid such a situation, we propose an approach in which the minimum required blockage time of each maintenance operation is extended with buffer time to compensate for the variation in maintenance duration. The details have been described in Section 3.

3. Mathematical model and solution approach

3.1. Mathematical model

In this study, we utilize a stochastic mathematical programming model to optimize the simultaneous scheduling of trains and maintenance activities of the track. The proposed model considers the assumptions and aspects described in Section 2.

As it was explained, we allocate buffer times to the blockage duration of each track segment under maintenance to cope with unexpected delays in the operations. The buffer time is considered as a stochastic variable with a given distribution function obtained from historical data. Figure 3 illustrates a typical normal distribution of variations in maintenance operations based on historical data. The buffer time is further determined based on the amount of confidence level (probability level) that the maintenance operation may exceed its minimum duration. Such confidence level is indicated by the hatch in Fig. 3. A high confidence level results in the buffer time being increased. However, a large buffer time represents an unused reserve time in the timetable and may cause the loss of available capacity. Therefore, adopting a well-balanced probability level is an important issue in determining the buffer time.

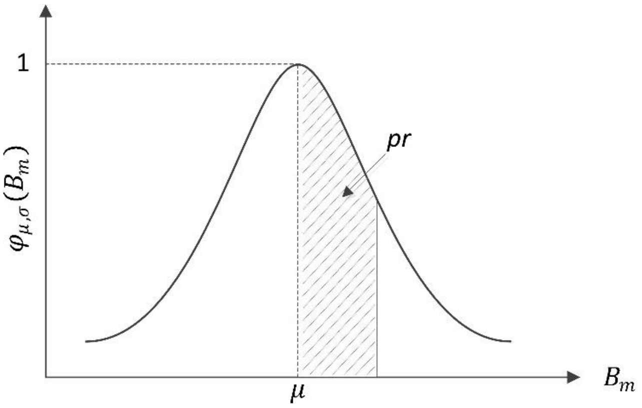


Figure 3: Determination of buffer time with a normal distribution

Another key feature of the proposed mathematical model in this paper is the use of a continuous-time horizon rather than the discrete-time intervals for simultaneous scheduling. In contrast to previous models [4,7,8,11], the results by the continuous-time horizon can be directly used in reality. In discrete-time models, the intervals are determined according to the viewpoint of users, and therefore the results may not have sufficient accuracy. Moreover, the division of the long-time horizon into small time interval length leads to very large combinatorial problems of intractable size.

The notation system of the stochastic mathematical programming model is provided in Table 2.

$$min\sum_{i\in I} w_i(TA_i^d - TD_i^o) + \sum_{j\in J} w_j(TA_j^d - TD_j^o) + \sum_{m\in M, p\in P}(TF_m^{(p),(p+1)} - TB_m^{(p),(p+1)}) \tag{1}$$

$$G.\alpha_{ij}^{(p),(p+1)} + TD_j^{p+1} \geq TA_i^{p+1} + h \qquad \forall p \in P, \forall i \in I, \forall j \in J \tag{2}$$

$$G.\left(1 - \alpha_{ij}^{(p),(p+1)}\right) + TD_i^p \geq TA_j^p + h \qquad \forall p \in P, \forall i \in I, \forall j \in J \tag{3}$$

$$G.\beta_{ij}^{(p),(p+1)} + TD_i^p \geq TA_j^{p+1} + h \qquad \forall p \in P, \forall i, j \in I \tag{4}$$

$$G.\left(1 - \beta_{ij}^{(p),(p+1)}\right) + TD_j^p \geq TA_i^{p+1} + h \qquad \forall p \in P, \forall i, j \in I \tag{5}$$

$$G.\gamma_{ij}^{(p),(p+1)} + TD_i^{p+1} \geq TA_j^p + h \qquad \forall p \in P, \forall i, j \in J \tag{6}$$

$$G.\left(1 - \gamma_{ij}^{(p),(p+1)}\right) + TD_j^{p+1} \geq TA_i^p + h \qquad \forall p \in P, \forall i, j \in J \tag{7}$$

$$TD_i^o \geq te_i^o \qquad \forall o \in O, \forall i \in I \tag{8}$$

$$TA_i^d \leq tl_i^d \qquad \forall d \in D, \forall i \in I \tag{9}$$

$$TD_j^o \geq te_j^o \qquad \forall o \in O, \forall j \in J \tag{10}$$

$$TA_j^d \leq tl_j^d \qquad \forall d \in D, \forall j \in J \tag{11}$$

$$TA_i^p + mns_i^p \leq TD_i^p \qquad \forall p \in P, \forall i \in I \tag{12}$$

$$TA_j^p + mns_j^p \leq TD_j^p \qquad \forall p \in P, \forall j \in J \tag{13}$$

$$TA_i^p + mxs_i^p \geq TD_i^p \qquad \forall p \in P, \forall i \in I \tag{14}$$

$$TA_j^p + mxs_j^p \geq TD_j^p \qquad \forall p \in P, \forall j \in J \tag{15}$$

$$P_r(TF_m^{(p),(p+1)} - TB_m^{(p),(p+1)} \geq t_m^{(p),(p+1)} + B_m) \geq pr \qquad \forall p \in P', \forall m \in M \tag{16}$$

$$TF_m^{(p),(p+1)} \leq tfl \qquad \forall p \in P', \forall m \in M \tag{17}$$

$$TB_n^{(p),(p+1)} = TF_m^{(p),(p+1)} \qquad p \in P, m, n \in M' \tag{18}$$

$$G.\theta_{mn}^{(p),(p+1)} + TB_n^{(p),(p+1)} \geq TF_m^{(p),(p+1)} \qquad p \in P', m, n \in M \tag{19}$$

$$G.(1 - \theta_{mn}^{(p),(p+1)}) + TB_m^{(p),(p+1)} \geq TF_n^{(p),(p+1)} \qquad p \in P', m, n \in M \tag{20}$$

$$TA_i^{p+1} - TD_i^p \geq \frac{l_{(p),(p+1)}}{v_{(p),(p+1)}} \qquad \forall p \in P'', \forall i \in I \tag{21}$$

$$TA_j^p - TD_j^{p+1} \geq \frac{l_{(p),(p+1)}}{v_{(p),(p+1)}} \qquad \forall p \in P'', \forall j \in J \tag{22}$$

$$TA_i^{p+1} - TD_i^p \geq \left(\frac{l_{(p),(p+1)}}{v1_{(p),(p+1)}}\lambda_{im}^{(p),(p+1)}\right) + \frac{l_{(p),(p+1)}}{v2_{(p),(p+1)}}\left(1 - \lambda_{im}^{(p),(p+1)}\right) \qquad \forall p \in P', \forall m \in M, \forall i \in I \tag{23}$$

$$G.\left(1 - \lambda_{im}^{(p),(p+1)}\right) + TB_m^{(p),(p+1)} \geq TA_i^{p+1} \qquad \forall p \in P', \forall m \in M, \forall i \in I \tag{24}$$

$$G.(\lambda_{im}^{(p),(p+1)}) + TD_i^p \geq TF_m^{(p),(p+1)} \qquad \forall p \in P', \forall m \in M, \forall i \in I \tag{25}$$

$$TA_j^p - TD_j^{p+1} \geq \frac{l_{(p),(p+1)}}{v1_{(p),(p+1)}}\left(\lambda_{jm}^{(p),(p+1)}\right) + \frac{l_{(p),(p+1)}}{v2_{(p),(p+1)}}\left(1 - \lambda_{jm}^{(p),(p+1)}\right) \qquad \forall p \in P', \forall m \in M, \forall j \in J \tag{26}$$

$$G.\left(1 - \lambda_{jm}^{(p),(p+1)}\right) + TB_m^{(p),(p+1)} \geq TA_j^p \qquad \forall p \in P', \forall m \in M, \forall j \in J \tag{27}$$

$$G.(\lambda_{jm}^{(p),(p+1)}) + TD_j^{p+1} \geq TF_m^{(p),(p+1)} \qquad \forall p \in P', \forall m \in M, \forall j \in J \tag{28}$$

$$0 \leq TA_i^p, TD_i^p, TB_m^{(p),(p+1)}, TF_m^{(p),(p+1)}, B_m \leq 1440 \tag{29}$$

$$\alpha_{ij}^{(p),(p+1)}, \beta_{ij}^{(p),(p+1)}, \gamma_{ij}^{(p),(p+1)}, \theta_{mn}^{(p''),(p''+1)}, \lambda_{im}^{(p''),(p''+1)} = 0 \text{ } or \text{ } 1 \tag{30}$$

In the following, the stochastic mathematical programming model is introduced. Objective (1) is composed of three parts. The first part minimizes the travel time of west trains for all ODs. This statement makes west trains run over railway lines with the maximum allowable speed and have the least dwell time in stations. Similarly, the second part minimizes the travel time of east trains. The third part of the objective function also allows maintenance operations to be carried out in the shortest possible time. The later part aims to adopt the optimal blockage duration of each maintenance activity. Constraints (2) and (3) prevent conflict of west and east trains in the same track segment. Constraints (4) and (5) prevent two east trains from entering the same block simultaneously, while constraints (6) and (7) make such prevention for two west trains. Constraints (8) and (9) determine the earliest departure time of west trains from their origins and the latest arrival time to destinations, respectively. Similarly, constraints (10) and (11) determine the earliest departure time and the latest arrival time of east trains, respectively. The minimum dwell time of west and east trains in stations is taken into consideration through constraints (12) and (13), respectively. Also, the most allowable dwell time in stations is considered by constraints (14) and (15). Constraint (16) accounts for the blockage duration of track segment (p,p+1) due to maintenance activity m∈M so that it would be greater than the minimum required blockage time extended by a buffer time with a given confidence level. This relation is the only probabilistic constraint that makes the model be of a stochastic type. Constraint (17) makes certain that the finishing time of the maintenance activities is followed according to the working hours of the crew. Constraint (18) causes some of the maintenance operations to be carried out successively to achieve the best quality of operations. On the other hand, Constraints (19) and (20) prevent the conflicts of two different maintenance operations in the same track segment. The maximum allowable speed of west and east trains in track segments that do not require any maintenance is given by constraints (21) and (22), respectively. The maximum allowable speed of west trains in track segments requiring maintenance before and after operations is determined by constraint (23). During maintenance operations, trains are not allowed to enter the track segments under repair. For this purpose, west trains are prevented from traversing track segments during maintenance activity by constraint (24), and they are enforced to wait until the operation is terminated by constraint (25). The maximum allowable speed of east trains in track segments requiring maintenance before and after operations is determined by constraint (26). Moreover, constraints (27) and (28) inhibit east trains from entering the track segment between the starting and finishing times of the maintenance operations, respectively. Constraint (29) specifies the time horizon of the related variables. Also, the integrality condition for binary variables is preserved by constraint (30).

3.2. Solution approach

In this paper, the proposed model is stochastic mathematical programming that is difficult to solve using ordinary solvers for medium and large-scale networks. The proposed approach of this study is to reformulate the chance constraint (constraint 16) to a deterministic one.

Generally, if the left-hand side (LHS) coefficients of a constraint in the optimization formulation are random variables, the chance constraint can be converted to a nonlinear deterministic form. The random variable in constraint 16 is the buffer time and occurs in the RHS of the chance constraint. Therefore, to convert the chance constraint to its respective deterministic equivalent according to the predetermined confidence level, it should be first rewritten as follows:

$$P_r\left(TB_m^{(p),(p+1)} - TF_m^{(p),(p+1)} + t_m^{(p),(p+1)} \le -B_m\right) \ge \alpha_m \quad \forall m \in M \tag{31}$$

To abbreviate Eq. 31, it can be again rewritten as

$$P_r\{g_m(X) \le \beta_m(\eta)\} \ge \alpha_m \quad \text{or} \quad P_r\{\beta_m(\eta) \le g_m(X)\} \le 1 - \alpha_m \quad \forall m \in M \tag{32}$$

where $g_m(X)$ stands for $TB_m^{(p),(p+1)} - TF_m^{(p),(p+1)} + t_m^{(p),(p+1)}$, and $\beta_m(\eta)$ for $-B_m$.

The deterministic equivalent of Eq. 32 can be determined if the distribution of $\beta_m(\eta)$ is known. According to historical data, it is assumed that $\beta_m(\eta)$ is normally distributed. Given the estimated mean and standard deviation for $\beta_m(\eta)$, Eq. 32 can be converted to

$$P_r\left\{\frac{\beta_m(\eta) - E_\eta[\beta_m(\eta)]}{\sigma_{\beta_m}} \le \frac{g_m(X) - E_\eta[\beta_m(\eta)]}{\sigma_{\beta_m}}\right\} \le 1 - \alpha_m \qquad \forall m \in M \tag{33}$$

where $E_\eta[\beta_m(\eta)]$ is the expected value of $\beta_m(\eta)$, and σ_{β_m} is the standard deviation of $\beta_m(\eta)$.

As $\beta_m(\eta)$ is normally distributed, the left-hand side (LHS) of the inequality inside the probability expression in Eq. 33 follows the standard normal distribution (mean=0 and SD=1). Therefore, the original chance constraint Eq. 16 is now tantamount to the following deterministic form:

$$\frac{g_m(X) - E_\eta[\beta_m(\eta)]}{\sigma_{\beta_m}} \le \Phi^{-1}(1 - \alpha_m) \quad \forall m \in M \qquad (34)$$

$$g_m(X) \le E_\eta[\beta_m(\eta)] + \Phi^{-1}(1 - \alpha_m)\sigma_{\beta_m} \quad \forall m \in M \qquad (35)$$

where $\Phi^{-1}(1 - \alpha_m)$ is the inverse function is of $\Phi(1 - \alpha_m)$, the standardized normal distribution evaluated at $1 - \alpha_m$.

Both Eqs. 35 and 16 have the same concept. Nevertheless, converting Eq. 16 to Eq. 35 makes the optimization problem easier to solve. It should be noted that if the random variable of the chance constraints has a distribution function other than the normal type, the related deterministic equivalent would be nonlinear. In a case where the random variable is normally distributed, the transformation results in linear deterministic constraints. The interested readers are referred to Vajda [12] and Watanabe and Ellis [13] for more details.

4. Result analysis

The implemented model is applied to a stretch of the southern railway line of RAI to verify the validity and reliability of the results. The examined stretch is a single line corridor that contains 14 track segments and 15 stations with a total length of 208 Kilometers. Figure 4 represents the railway line and block sections. 13 passenger and freight trains are scheduled in each direction per day between Andimeshk and Dorud Stations. According to the available inspections, the second category of maintenance operation (tamping and ballast regulating) is planned for track segments No. 8 between Tale-Zang and Tang-e-Panj Stations, as highlighted in Fig. 4. By such maintenance operation, ballast regulating should be successively operated after tamping.

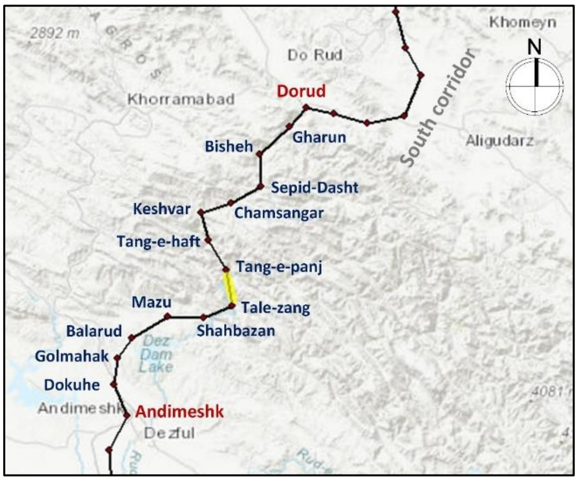
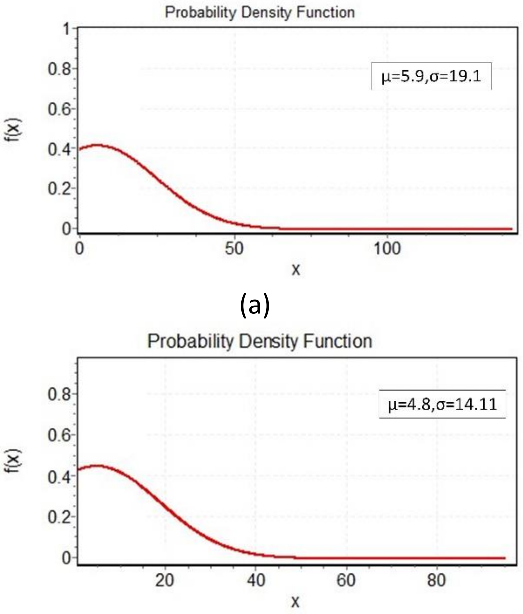


Figure 4: The stretch of the railway line.

The available data of maintenance operation for this track reveals that the delay might occur for each type of maintenance category. Therefore, it is required to consider a buffer time for each maintenance type. According to historical data, the delays that occurred for tamping and ballast regulation follow a normal distribution, as displayed in Fig. 5.



(a)

(b)

Figure 5. Distribution function of (a) tamping and (b) ballast regulating.

The time window for the scheduling is considered for a whole day (1440 minutes), and the time 6 AM is set as the beginning of the trains' timetable. Due to the lack of special maintenance facilities and the limitation of working hours during night hours, maintenance operations shall be terminated at 2 PM (500th-minute). Furthermore, the maximum speed of trains on the block section requiring maintenance is reduced to 45 km/h due to the poor quality of the track. After the operations, however, the speed limit returns to a normal state. Table 3 summarizes the inputs of the model for the case under study.

Table 3. Inputs of the model for the case under study

Parameter	**Value**
Maximum speed at the regular track segments	75 km/h
Maximum speed at the track segments requiring maintenance before the operation	45 km/h
Maximum speed at the track segments requiring maintenance after the operation	75 km/h
Importance coefficient of passenger trains compared to freight trains	2
Minimum required blockage duration for tamping	100 min
Minimum required blockage duration for ballast regulating	90 min
The safety time interval between two successive trains	5 min
Confidence level in the chance constraint	50%

The case was analyzed through the proposed stochastic optimization formulation to obtain the simultaneous scheduling of trains and maintenance operations. The confidence (probability) level in the chance constraint (Eq. 16) is set to 50%. It represents the confidence level with which the proposed buffer time covers the probable delay of that maintenance operation. Figure 6 illustrates the simultaneous scheduling of trains and maintenance operations in such a case. According to this graph, the blockage time needed for tamping and ballast regulating of the track segment No. 8 is obtained 106 and 95 minutes, respectively, which implies that the total buffer time of 11 minutes is required for maintenance operation of that segment. Figure 6 also shows the maintenance operation in track segment No. 8 should be started at 9 AM (120th minutes) to obtain the minimum travel time of trains and maintenance operation.

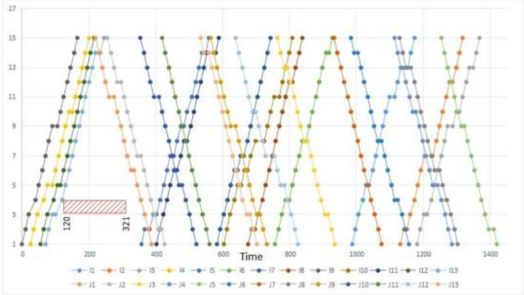


Figure 6. Timetable of trains and maintenance operation with 50% confidence level

Considering different values of confidence level in the chance constraint (Eq. 16) leads to an over-optimistic or under optimistic estimation for buffer time of maintenance operation. Table 4 shows the required buffer time for each confidence level in addition to the start time and end time of those operations.

Table 4. Variation in maintenance duration according to confidence level

Confidence level (%)	50	60	70	80	90	95
Start time of tamping (min)	120	120	115	235	115	119
End time of tamping (min)	226	231	231	357	246	257
Buffer time of tamping (min)	6	11	16	22	31	38
Start time of ballast regulating (min)	226	231	231	357	246	257
End time of ballast regulating (min)	321	329	333	464	358	375
Buffer time of ballast regulating (min)	5	8	12	17	22	28

A challenge often faced by the decision-maker is how to consistently select an acceptable buffer time for each maintenance operation. By varying the probability level in the chance constraint Eq. 16, we can obtain a range of values for the buffer time of each maintenance operation. Figure 7 shows the buffer time related to each confidence level. As can be seen, the higher the confidence level is considered for each maintenance type, the greater the buffer time is assigned to it. Meanwhile, the probability of delay occurrence (columns in Fig.7) reduces with increasing the buffer time for each maintenance type. For example, when the buffer time for tamping is 11 minutes, the probability of unexpected delay in tamping operation is 40%, while with a 38-minute buffer time, it is only 5% possible that a greater delay occurs during that operation. A higher confidence level declares that the obtained buffer time will cover more delays that occurred during a maintenance operation. This is especially remarkable in situations where every slight increase in blockage time causes significant delays or even cancellation of the trains. Therefore, a higher confidence level might be considered for such cases.

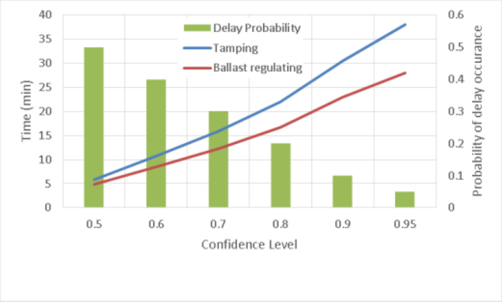


Figure 7. Selecting the appropriate buffer time based on the confidence level.

As discussed above, assigning a buffer time to the blockage time of each maintenance operation prevents delay propagation if the operation occurs with a delay. However, this often causes an increase in capacity consumption which is a problem for heavily utilized lines. In this study, we increased the number of trains in the current timetable to evaluate the effect of compressing timetable on the buffer time of maintenance operation. The analysis reveals that when the number of trains increases to 15 trains per direction, the required buffer time as per confidence level is the same. For example, when there are 15 trains or fewer in each direction, a total buffer time of 11 minutes is required for maintenance operation at a 50% confidence level. However, the implementation time to solve an optimal timetable dramatically increases with increasing the confidence level and the number of trains. This is when the number of trains increases to 16 pairs, a feasible timetable cannot be found. It implies the fact that the unused time reserves by considering a high confidence level at a compressed timetable makes it infeasible. Therefore, it is required that an appropriate confidence level should be addressed by the decision-maker according to the compression of the timetable.

5. Conclusions

This paper presents an optimization approach for the simultaneous scheduling of trains and maintenance operations for single-track lines. The proposed model is based on stochastic mathematical programming, which considers the uncertainty in blockage time of track due to unexpected delays in maintenance operations. The model has been solved by converting the chance constraint to a deterministic one when the distribution function of expected delays is available by the historical data. The proposed approach has several properties that distinguish it from previous ones. First, it considers the requisite practical aspects for simultaneous scheduling of trains and maintenance operations such as (i) compulsory sequence of some maintenance activities, (ii) allowable speed of trains before and after the operation, (iii) crossing and overtaking of trains at stations, (iv) departure and arrival time of trains, and (v) working hours of crew. Next, the study considers a continuous time horizon for scheduling of operations and trains instead of discrete intervals, which has been addressed by previous studies because of simplicity. Furthermore, the unplanned delay of operation is taken into the scheduling by including a stochastic buffer time to the minimum required blockage duration of each maintenance type. This improves the reliability and on-time performance of the timetable by decreasing delay propagation and therefore increasing the timetable robustness.

In this paper, the results of applying the model on the south corridor of the Iranian Railway have been reviewed and analyzed. As the buffer time is addressed to be a random value, different confidence levels were considered for the related chance constraint. The sensitivity analysis on the confidence level shows that considering higher levels results in greater buffer time for each maintenance type. With increasing the confidence level, however, the probability of a delay greater than the obtained buffer time will be reduced. In other words, the scheduling by a higher confidence level is expected to be more robust against unplanned delays in maintenance operations. It should be noted that the buffer time consumes the timetable capacity, which is a problem for heavily utilized lines. Our analysis shows that when the number of trains increases in the current timetable, the obtained buffer time for a specific confidence level does not change. However, with the compression of timetables, feasible scheduling can hardly be obtained. It implies the fact that the unused time reserves by considering a high confidence level at a compressed timetable makes it infeasible. Then, the decision-maker should select the appropriate confidence level regarding the compression of the timetable and the probability of unexpected delays.

There are several directions to be considered for future research on the problem of simultaneous scheduling of trains and maintenance operations. Firstly, the buffer time for trains can be accommodated into the timetable to compensate for delays in main trains. The proposed method is to use a probabilistic constraint for the speed limit of trains. The second direction is to extend the model to consider the capacity of stations for arrival and departure trains. Furthermore, some maintenance equipment should be located in major stations that require scheduling to depart to those track segments requiring maintenance. Such issues have not been considered in the current problem and can enhance the operational aspect of the model.

Declaration of Conflicting Interests

It is acknowledged that no authors of this manuscript have any actual or potential conflict of interest, including any financial, personal, or other relationships with other people or organizations.

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