



## Non-dominated Sorting Bees Algorithm for Multi-Objective Train Speed Profile Optimization

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### ABSTRACT

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Considering various factors like energy consumption and travel time, speed profile optimization is regarded as a non-linear, complex multi-objective problem. In this paper, a novel approach based on the Bees algorithm (BA) is proposed to solve train speed profile optimization. In this approach, BA is modified as a non-dominated sorting BA (NSBA) which produces a population that conducts a semi-stochastic search algorithm to determine critical points for acceleration changes forming a raw template as a trajectory. Afterward by local optimizations, the trajectory is amended to meet ending requirements. The simulation results indicate on effectiveness and excellence of the proposed method.

## 1. Introduction

The train speed profile is of great importance since it deals with major factors of train performance as journey time, capacity analysis, train schedules, and energy consumption. A significant bulk of research is available on the optimization of speed profiles and related models. In general, train optimization is divided into two types: coast control and general control. In models based on coast control, the optimization of the train speed profile is based on an adjustable margin which can be used to perform the coasting. In this model, various methods have been developed and applied to calculate the costing points and the corresponding number of points. In general control mode, the speed profile optimization is

determined by using points for acceleration, coasting, and braking.

Determining the optimal profile is possible in various ways, divided into three categories: analytic, numerical, and evolutionary methods. One of the early approaches for speed profile optimization is the analytical methods in which, based on the optimal control theory, the problem is solved via Pontryagin maximum principle [1-3]. This method leads to an exact optimal solution; however, it requires a well-detailed model. In numerical methods such as dynamic programming [4, 5] or the Lagrange method [6], the model requirements are less strict, but they have a palpable calculation mass which is the main drawback for such methods. More recently the evolutionary algorithms have proved to be a promising strategy in optimization problems in different fields. There is an assertive number of

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evolutionary algorithms developed for speed profile optimization as genetic algorithm [7], tabu search [8], and ant colony optimization [9]. These algorithms have the least requirements for system modeling; however, the convergence of the solutions is not guaranteed, and further studies are needed to approve the solutions. This shows a major trade-off in three methods between model complexity and optimality assurance. In this study, we discuss optimal train trajectory in the framework of the Bees algorithm (BA). The input is the altitude of the rail line as well as the train characteristics, and the output is the optimal speed profile. As the novelty of this paper, the BA is modified as non-dominated sorting BA (NSBA) for 2-dimensional optimization to satisfy the time and energy cost functions. The output as Pareto front is clarified based on practical concerns and convergence to choose the most elegant solutions for the speed profile.

The rest of this paper is organized as follows: the model of the train and the root information is described in section I. The methodology describes BA structure and NSBA in section II. Simulation results and further discussions are expressed in the next section, and eventually, concluding remarks are drawn in the final section.

## 2. Vehicle Model and Root Information

The movement of the train in the longitudinal direction is determined by several forces, including traction force, braking force, and resistant forces. The resisting forces are divided into two categories: the basic resistant forces and the resistant forces related to the line characteristics. Fig.1 illustrates the applied forces to the train. The basic resistant forces originate from the friction and are constantly present and effective in the movement of the train on the rails; hence they are called basic resistant forces. The basic resistant forces can be determined by the Davis equation presented in

Eq.1 [10], in which  $R_{basic}$  is the basic resistance  $x$  is the distance, and  $A, B, C$  coefficients are mainly derived from experimental measurements.

$$R_{basic}(x) = A(\dot{x})^2 + B(\dot{x}) + C \quad (1)$$

It should be noted that the basic resistant forces are correlated to the movement of the train on a surface without slope, parallel to the horizon line, and without curvature or any arcs. The existence of curvature and slope with different angles in the structure of the line leads to the other group of resistant forces called line resistance. The grade resistance is related to the movement of the train with a certain weight on the sloping surface, which forms a certain angle with the horizon line. Fig.1  $F_w$  represents the grade resistance. The equation for the grade resistant force is given in Eq.2 [10].

$$F_w = Mg \sin(\alpha(x)) \quad (2)$$

Where  $M$  is the train weight,  $g$  is the gravitational acceleration, and  $\alpha$  is the slope angle. According to this relation, it can be seen that the grade resistance, unlike the basic resistant force, is not related to the speed of the device.

One more complex resistive force in the train motion is the resistive curve force. This force is measured per ton of train weight per degree of curve. There are various and sometimes contradictory theories about this force [10]. Therefore, experimental measurements are the main basis for this force reporting. Centrifugal theory is one of these theories. For a better understanding of this force, it should be noted that the wheels of a train always tend to move in a direction perpendicular to the axis of rotation, and this tendency leads to severe pressure of the outer wheels of the train on the outer side of the rail, which is the source of this resistant force and

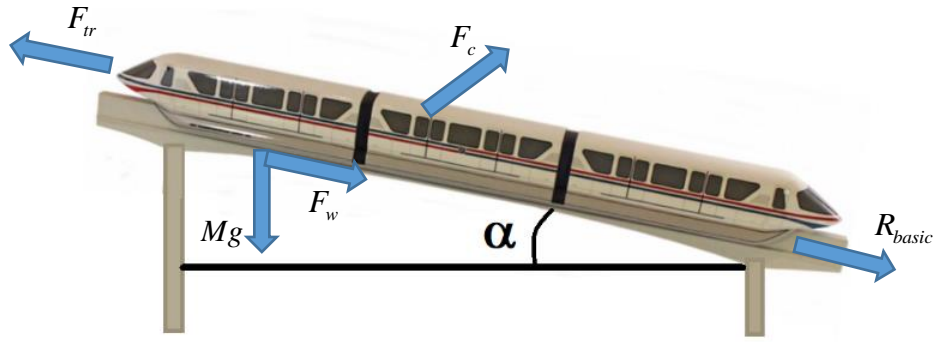


Figure 1. The free-body diagram of the train on the inclined plane.

a considerable part of locomotive power is spent to counter this force. Eq.3 is an experimental relation based on measurements for this force.

$$F_c = k_c \frac{M}{r(x)} \quad (3)$$

Where  $M$  is the train weight,  $r$  is the curve radius, and  $k_c$  is an adjustment coefficient. According to Eq. 3, this force, like the grade-resistant force, is independent of the speed of the train. Finally, the equation of motion of the train based on the described forces and according to Newton's second law is as follows:

$$\ddot{x} = \frac{1}{M} [F_{tr} - R_{basic} - F_c - F_w] \quad (4)$$

Where  $F_{tr}$  is the traction force, and  $\ddot{x}$  is the vehicle acceleration. Using Eq.1 to 3, Eq.4 can be recast as:

$$\ddot{x} = \frac{1}{M} \left[ F_{tr} - (A(\dot{x})^2 + B(\dot{x}) + C) \right. \\ \left. - k_c \frac{M}{r(x)} - Mg \sin(\alpha(x)) \right] \quad (5)$$

Note that for a realistic simulation,  $M$  must be set as the effective mass. Finally, based on the relation between tractive power and traction force, the required power is obtained by:

$$P = \frac{F_{tr}}{\eta} \times (\dot{x}) \quad (6)$$

Where  $\eta$  is the efficiency of the power conversion. This study is conducted on the Qom monorail line between the stations of Payaneh and Farhangsara. The altitude profile and the speed limits of the route are illustrated in Fig.2. In the next section, we try to find the optimum speed profile for the train trajectory with a modified Bees Algorithm. This optimization is objected to minimize energy consumption as well as time travel. Since the problem conditions contrast one another, a Pareto front can be expected.

### 3. Methodology

Under this section, the applied method is described. First, the BA is discussed in detail, and then the NSBA is cast.

#### 3.1. Bees Algorithm Basics

Define Bee algorithm is one of the search algorithms based on swarm intelligence. This algorithm imitates the behavior of honey bees in nature [11]. In this algorithm, a local search is done along with a global search to select the optimal answer from various solutions. In nature, a certain number of bees randomly search

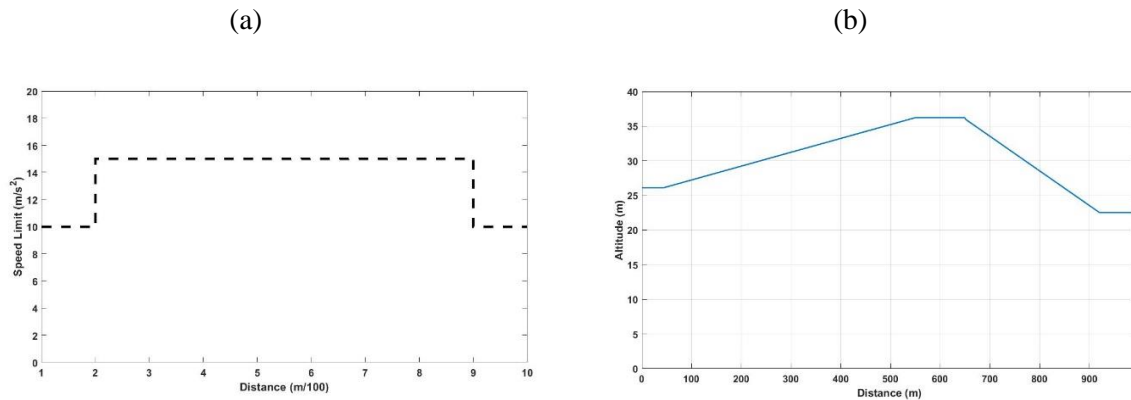


Figure 2. (a) The speed limit profile and (b) the journey altitude.

for food sources around the hive, and when they return to the hive, bees that have found high quality and acceptable food sources perform specific moves called Waggle dance on the dance floor where other bees are doing the same to communicate and inform the hive about the found food source. Notice that if the bees find a food source that does not satisfy quality conditions, they refuse to dance and look for other sources, so only bees that have returned to the hive from the desired food sources perform the waggle dance. Based on the dance performed, bees who are watching the dance of others select a food source based on the information obtained. The number of bees that are attracted to a food source is based on the quality that the scout bee reports from that food source. By this means, the hive is informed about the quality of the food source as well as its location from the aspect of the scout bee. However, the scout bee may not be completely accurate about the location. Furthermore, the looker bees have their judgment about the dances, and this is the critical point where local optimization occurs. Each of these recruited bees goes to a different location but in the neighboring of the reported source. This concept

is illustrated in Fig.3. This fact helps to amend the source or even find a better one. This is one of those times that inaccuracy comes in handy. In other words, each bee, by looking at the dance of another bee, makes its perception of the location of that food source, so it is possible that two bees have different perceptions, and neither of them goes exactly to the source reported by the first bee. This makes a neighborhood space around each source. Based on this approach, the Bees algorithm is performed as follows:

- Generate initial responses and evaluation.
- Allocate a higher number of improvement chances to better solutions.
- Create a new solution based on the allocated number of acceptable solutions.
- Evaluation of new solutions.
- Compare all solutions and choose the best one.
- Random search to find alternative solutions for unselected sites.
- Save the best answer.
- Repeat until the ending conditions.

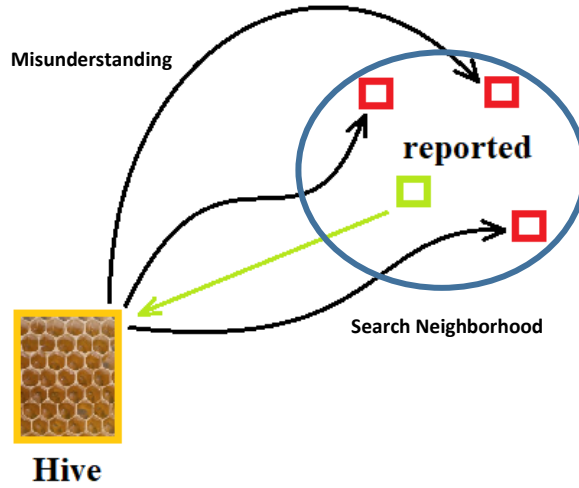


Figure 3. The searching paradigm for a food source. The green square shows the reported source location. The red squares are the locations that looker bee referred to as the reported source.

### 3.2. Non-dominated Bees Algorithm

The BA can be used for multi-objective optimization by using the concept of domination. A multi-objective BA is proposed in [12]. In this paper, we modify the BA using non-dominated sorting for the first time. BA does not generally produce a population. The newfound solutions are only compared with their origin only when one of them is kept. However, a weaker solution in a neighborhood might have the potential to produce better solutions in the next generations. Therefore, we modified BA to produce a population from older solutions. The elderly solutions and their generations are compared with all the others (not just themselves) and will have some chance to produce population even if their origin is a better solution. This modification helps the algorithm to have a greater search power, both global and local. Initially, we define the domination in Eq.7 as below:

$$(X \prec Y) \text{ if } \begin{cases} f_1(X) \leq f_1(Y) \\ f_2(X) \leq f_2(Y) \\ \dots \\ f_n(X) \leq f_n(Y) \end{cases} \quad (7)$$

In which  $X$  and  $Y$  are two solutions for the multi-objective problem and  $f_i$  is the cost function in the  $i_{th}$  domain. In the non-dominated sorting bees algorithm (NSBA), instead of substituting the best solution (based on cost functions and crowding distance), we produce a population. The updated term remains as it was in BA shown in eq.8.

$$x_{new} = [x_{1,new} \quad x_{2,new} \quad \dots \quad x_{D,new}] \quad (8)$$

$$x_{i,new} = N(x_{i,old}, \sigma^2) \quad (9)$$

Where  $N$  is the normal distribution, and  $\sigma^2$  is the variance. In the zero stage, the early answers from initialization are sorted based on domination set and crowding distance. Upper 50% is the accepted answer, while the rest must be substituted in the next iteration. Then the accepted answers are divided into two groups of elite and non-elite. We use Eq.9 to produce a new population based on the selected solutions. However, the elite answers get more capacity. In other words, onlooker bees are sent to elite food sources, but none of the newfound sources is replaced by the other at this stage. The old and new solutions merge, and only upper  $(50 - \delta)\%$  are allowed to go to the next

iteration. Note that in each iteration, the population or the list of food sources grows since an individual accepted solution produces at least more than a single solution. Therefore, the parameter  $\delta$  is defined as the acceptance rate

limit to adaptively control the population and increases as the population does. This helps the algorithm to remain dynamic until the ending condition reaches. The block diagram of NSBA is presented in Fig.4

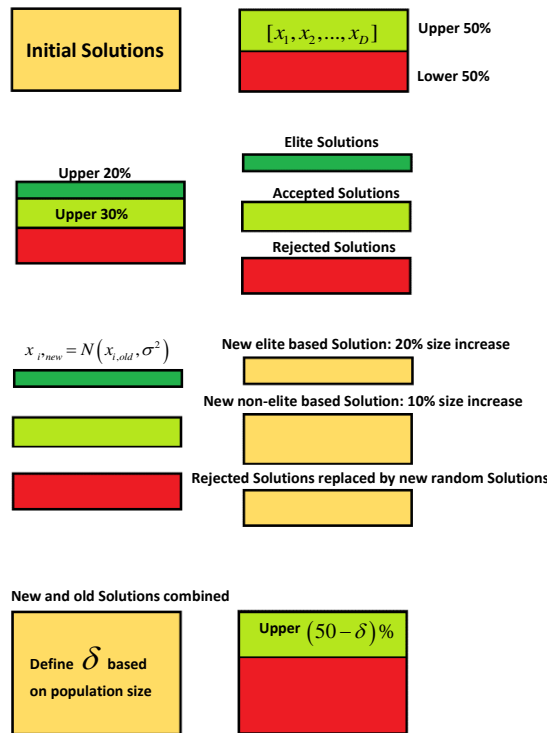


Figure 4. The block diagram of NSBA.

#### 4. Results and Discussion

The simulation results are drowning in this section. The proposed NSBA was utilized to determine the optimum trajectory. The journey time versus energy cost curve is compared in iterations. These curves are shown in Fig.5. It

can be seen that after an adequate number of iterations, the solutions have converged logically to optimality. This shows the capability of NSBA in solving the speed profile problem.

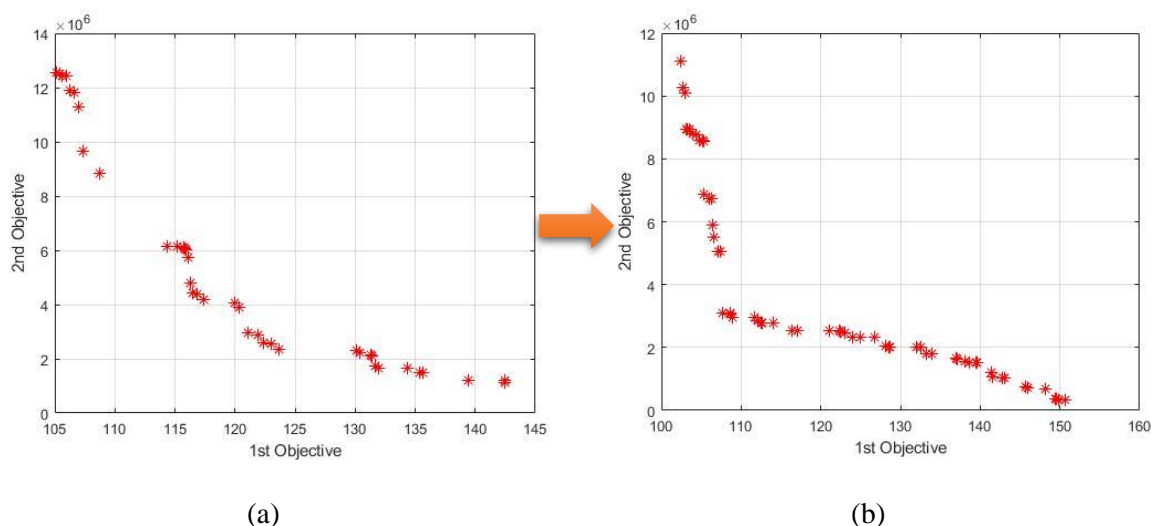


Figure 5. The Pareto front for NSBA after (a) 6 and (b) 60 iterations.

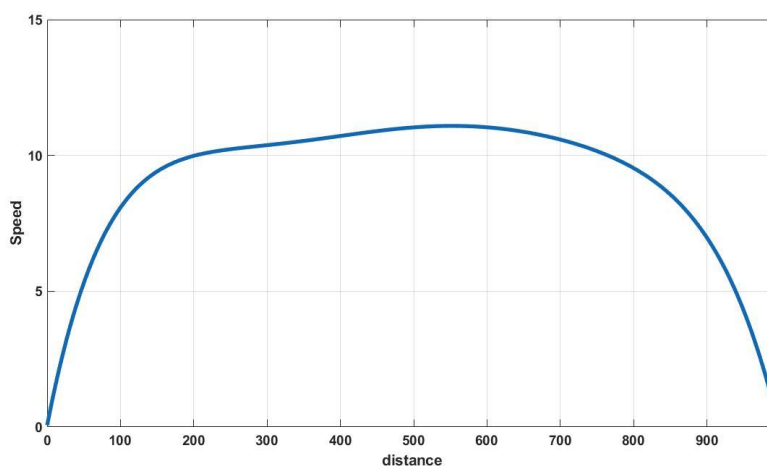


Figure 6. A candidate solution for the optimal trajectory.

In the final result, as shown in Fig.5 (b), the travel time varies from 105s 152s within the energy consumption of interval from  $0.2 \times 10^6 j$  to  $11.2 \times 10^6 j$ . Corresponding to each point in the Pareto front, a speed profile can be recommended. As an example, Fig.6 represents the 27<sup>th</sup> point in the Pareto front. The cost matrix is as  $[1.423920165667234e+02; 1.435969211075732e+06]$  where the first element is the time cost and the latter is the energy cost.

### 5. Conclusions

In this paper, the Bees algorithm was introduced for train trajectory optimization. BA was modified as NSBA for the first time to adapt to the multi-objective problem and enhance both global and local optimizations. The performance of the suggested networks was approved after early iterations and converged to the optimal solution. The results of the simulations showed that NSBA could solve the speed profile optimization problem with a wide range of costs. As future works for this study, considering the multiple train trajectory, optimizations, and

comparison with other meta-heuristic algorithms is recommended.

### Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

### List of symbols

The list of symbols in this paper is as follows:

$A$	Coefficient of 2 <sup>nd</sup> order term in Davis equation
$B$	Coefficient of 1 <sup>st</sup> order term in Davis equation
$C$	Constant term in Davis equation
$F_c$	Curve resistance
$F_{tr}$	Traction force
$F_w$	Grade resistance
$g$	Gravitational acceleration
$k_c$	Curve resistance coefficient
$M$	Train weight
$N$	Normal distribution
$x$	Position of the train
$x_{old}$	Solution of the previous iteration
$x_{new}$	NSBA new solution

### Greek symbols

$\alpha$	Slope angle
$\sigma$	Search Variance
$\eta$	Efficiency of the power conversion
$\delta$	Adaptive acceptance rate limit

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