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A Study to Predict the Development of Wear for the Wheel/interface of a Railway Track

Temitope Matthew Omojola[1], Sunday Ayoola Oke[2*]

[1,2]Department of Mechanical Engineering, University of Lagos, Lagos, Nigeria

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A B S T R A C T

The train transport is often confronted with greatly repeated changes in scope, vehicle travel well-being and transport safety because of the adjustments of wheel and rail profiles. These problems are frustrating and threatening. Thus, there is a requirement for an approach to predict the wear of wheel/rail to tackle these problems. A new theoretical ground is broken through the build-up of a modified structure motivated by dimensional analysis to predict the growth of the wheel profile by drawing on Li et al.'s seminal work on wheel profile. The principal difference between the current research and Li et al.'s work is the incorporation of additional wear profile parameters of fatigue strength and design factor. A framework is then developed for damped and undamped vibration circumstance with simulation results. The suggested approach could comparatively predict the needed parameters with the advantage of a wide scope of parametric coverage. The novel element of the paper is the unique way of predicting the desired parameters in a wider range of parametric observations. The probable use of the method is the state functional railway corporations in developing countries.

1. Introduction

1.1 General

For decades of consideration in determining the contributions of the various modes of transport, research on train transportation has distinguished it as a foremost method of economic and technical enlargement in both developed and developing countries. In examining these studies and comparing the available research information with the water ways mode of transportation, Berqquist and Woxenius' [1] overview of the area suggest that train transportation is an extremely important means of transportation to augment the extent of neighborhood transport transmission. This structure positions the economics of train transport with respect to two nouns-profitability and development of research scholars to direct attention to these areas. Although it is appreciated that such a direction of research interest has been useful in stating a plan of research for the area, it is declared here that it has restricted attention devoted to the technical procedures that comprises the area of train transportation. To elevate interest in research that studies more rigorously the technical concerns linked to train transport economic issues, this paper is interested in developing a new theoretical framework to predict the growth of the wheel profile following Li et al.'s [2] idea that conceptualizes the wear of the wheel/rail profile to tackle the subsequent problems associated with train transport economics: rail-maintenance cost, track's loading scope, vehicle travel well-being and transport safety because of the adjustments of wheel and rail profiles.

It is surprising that in spite of the overwhelming evidence on wear of wheel profiles as well as the richness of motivations by

*Corresponding author
Email address: sa_oke@yahoo.com

train stakeholders, train transport research has paid very insignificant attention to the problem of wheel/rail adjustments of profiles as a result of wear. This problem is frustrating, disturbing and threatening. Within the framework of the development (expansion), concept discussed earlier in this paper is an approach called the adoption of a technological assignment (TA) also referred to as technological mission. The TA is meant to direct the attention of nations to and propel contemporary technologies of examining, and controlling wheel profiles for railway wear regulation. For example, as noted by the government of India, technological assignment will attain the objectives related to the set missions as well as offer momentum to railway technological undertakings cross-wisely of the whole country. While the TA approach is commendable and large-scale in nature, an alternative approach is the predictive attempts at solving the problem. Taking a wider perspective would indicate that train transport research ought to direct attention on new solutions and new theoretical ground should be broken to build up modified structures, for example, drawing on Li et al.'s [2] seminal work on wheel profile. This particular goal is achieved in the current research. The principal differences between the present study and Li et al.'s [2] work are as follows. Here, a new incorporation of additional wear profile parameters of Fatigue strength and design factor. A framework is then developed to compute the wear volume in a damped and undamped vibration circumstance. Furthermore, simulation results are presented to reflect the practicality of the developed framework.

1.2 Problem Definition

Research on wheel/rail had been instigated by a number of classical presentations for decades. A strong attention of such studies was on the examination of wheel profile wear and its outcomes and this has revealed the necessity for more investigations in the area. In fact, the progressively declining economy of developing countries has made more investigations compelling. Consequently, in order to attain this goal, it is essential to critically examine some classical studies for radial improvements. In this respect, the seminal study by Li et al. [2] plays a central role in the current study. The investigation presented novel representations for predicting the development of the profile of the railway wheel as a result of wear. To attain the goal of the current research, it is essential to note what critical parameters are missing in the study and fix such parameters for the enhancement of the model. In a literature study, it was found that the surface fatigue strength and the design factors are the crucial missing parameters in the Li et al.'s [2] model. The result of this absence of essential parameters is that wrong evaluations and interpretations may be made. Although Li et al.'s [2] model advances research and helps in building the research area of the development of wheel profiles in railways as a result of wear, new knowledge is critically needed to advance the field. So, it was found out that at present, there is no literature source that exhibits this type of framework desired in this study. The omission of such representation in literature could be largely due to the fact that an integration of pre-operational concepts (i.e. design factor) and operational factor (i.e. surface fatigue strength) is missing among research till date. Besides, probably there is no pressure compelling to advance knowledge than now that the economy is biting. Thus, the omission of a framework for predicting the development of the wheel profile for railway when the surface fatigue strength and the design factors are incorporated into the Li et al.'s [2] model was contemplated as a research problem which must be solved in this work.

Based on the problem formulation in the previous section of this paper, the following ideas are pursued in the current research. A first step is taken to investigate the dynamics of the wheel/rail mating parts. Based on this, an enhanced model of Li et al.'s [2] model is pursued such that additions to incorporate wear profile parameters, for instance surface fatigue strength and design factors. After a successful development of a new model, a representation will be developed such that the wear volume could be calculated for cases of damped and undamped vibration. Given the success in this, simulation of the same is pursued to reflect the practicality of the developed model. A final step is to draw inferences from the simulation experimentation.

The scope of the study has orientation about the following issues:

- Extend Archard's wear model under the concerns of surface fatigue strength and design factor.

- Compare the current model with Li et al.'s [2] wear model using data obtained from Li et al.'s [2] and simulated data.
- Use the regression analysis to predict the wear volume to be obtained under surface fatigue strength and design factor and also in a damped and undamped vibration circumstances of the wheels.

1.3 Novelty and Paper Contributions

1.3.1. The novelty of the formulated

The new features of the present investigation could be detailed here from many viewpoints. First, a key characteristic is the buildup of a unique structure that connects the principal features of Li et al.'s [2] framework with two important parameters of surface fatigue strength and design factors. This has two dimensions of thought. From one dimension, the external connection of the former structure is enhanced as the surface fatigue strength parameter accounts for the interactive ability of the framework with the immediate environmental impact (physical). The other dimension is pre-operational in nature, accounting for a revision in the design, and suggesting an incorporation of some design factors into the model for more robust evaluation of the wheel profiles development. This elegant perspective is meant to capture the evaluative framework in a more realistic manner, leading to a practical visualization of the development of the wheel/rail profile from the perspective of wear. Consequently, the path taken by this contribution is toward wheel/rail operational enhancement since the information provided by the research is crucially necessary in design revisions by design engineers and new wheel/rail structural development and monitoring. It is interesting to note that the current research promotes knowledge and adds to the bank of knowledge on wheel/rail profile development monitoring and control. It adds to our knowledge of how material properties for the wheel/rail become very important in railway engineering practices. In fact, the design factors provoke engineers to revisit the foundational design principles, techniques and technologies to embrace innovative means of enhancing the wheel/rail profile through design. This as well has the advantage of elevating the competence of the design engineers in the skillful design of wheel/rail structures. In sum, the novel features of this research is developed en route to the production of railway industrial activities associated with the attainment of heightened outcomes as against less attractive ones.

1.3.2. Contributions

The contribution of this article is relevant to the railway wheel profile literature. The paper contributes through an open representation of form of wheel/rail profile development scheme. Despite the increasing economic impacts on the railway industry and the seemingly reduced funds to pursue the expected operational activities, the scientific approach to enhancing the evaluation of the wheel/rail profile has not been completely understood. The put forward framework begins by tackling this literature gap by declaring the two principal parameters that could enhance capturing the development of the wheel/rail profile in prediction. A second contribution of the research is the development of a representation that is utilized to compute the wear volume of the wheel/rail in a situation of damped and undamped vibration. Third, a contribution while this was discussed in the research is the successful demonstration of the manner to simulate the volumetric measurement of the wear to reflect the practicality of the developed model.

2. Literature Survey

2.1 The Wheel/Rail Literature

As the research topic indicates, an exercise of prediction of the progression of wear that occurs at the wheel/rail interface is carried out in an effort to gain insight into the mechanism of wear formulation for the wheel/rail system. This prediction is at the convenience of railway engineers and scientists to direct technical effort towards the required work content due to the anticipated volume of wear. It will also aid in the economic planning for the wheel/rail maintenance and development project. Consequently, a re-orientation of the developed Li et al.'s [2] representation is essential to successful achievement of the aforementioned goals of practicing engineers and managers of the railways industry. In the field of the wheel/rail literature, the aspect that ought to be emphasized so as to achieve the practicing goals is the qualification of surface fatigue strength. Also, the design factors ought to be appraised.

This appreciation of this fact reveals that there is requirement to build a new model incorporating surface fatigue strength and design factors in the integrated framework involving Li et al.'s [2] proposed. Following this background, review of literature as revealed in this section was conducted.

A number of research papers were largely focused on the development of models of wear concerning the wheel/rail system. An interesting study was contributed by Leary et al. [3] with the main focus on the buildup of two novel methods, namely a mean worn wheel profile as well as profiles rooted on increasing the size of rail shapes. Still on model development, a multi body schematic representation for the rail vehicle was show cased in Rezvani et al. [4]. The important features of the model is the consideration of the scheme as an elastic body, the appraisal of novel and work put profiles with suitable boundary situation to account for the wheel/rail connections. The concerns of Fujioka and Iguchi [5] was modification of wheel/rail profile with due consideration for gear's involute curve and the term non-linear dynamic representation was given to the contribution. The model was useful to appraise load circumstances of the unevenness of the track. The interest of Shevtsov et al. [6] was the wheel profile's design with the enhancement of the wheel/rail interface in mind. It is one of the very few studies that considered wear under the conditions of rolling contact fatigue subjected the most advantageous criterion rooted in the radii difference procedure.

2.2 Review on Surface Fatigue Strength and Design Factors

After reviewing the above mentioned literature, two principal literature sources emerges as critical for inclusion in the current literature review. These are the study by Shigley and Mischke [7] that focused on fatigue in metals and Norton [8] that also addressed fatigue. The other aspect of design factor in design was also incorporated in the book presentation by Shigley and Mischkle [7] and this two mentioned literature sources are critical to the advancement of the topic pursued in the present research.

In treating the issue of fatigue in metals, Shigley and Mischkle [7] observed the failure of metals when influenced by stresses behaving in either regular or irregular manner. A further insight by the author indicated that real utmost stresses were lower than the material's strength as well as regularly lower than the strength at the yield point and the failure is referred to as the fatigue failure. The buildup of fatigue failure commences by means of a little crack. At the elementary level, thin crack cannot be noticed by eyes because of its little nature, and also challenging to notice it through the X-ray. However, the authors asserted that the crack attains a discontinuity point within the material as it experiences transition in size and area. The authors further asserted that the moment a crack is initiated the impact of concentration of stress turns to be high and then the rapid progression of the crack is experienced. As the stress area shortens in size. The stress grows in dimension pending the period at which the other material suddenly fails. Norton [8] stated that the majority of fatigue failures observed in service as well as under controlled laboratory test are principally the result of poor design and machining practice. The introduction of critically cylindrical stressed parts may give rise to crack nucleation and propagation, so that ultimate failure occurs. Metallurgical factors related to poorer fatigue property are the presence of retained austenite in hardened steels, the presence of flakers or sharp inclusions in the microstructure, and the treatments which induce preferential corrosive grain boundary attack. Another aspect treated in Shigley and Mischke [7] is the design factor, in which the author stated that the concept of stress presented the opportunity to define the design factor from the perspective of stress. The authors further asserted that with the availability of testing machines where particular modes of failure could be induced, design factors could be from the viewpoint of stress and the associated strength.

2.3 General Studies

Li et al. [2] developed an approach to predict the wheel profile wear for enhancing an existing representation to determine the railway vehicular wheels' wear moving on tracks characterized by sharpness and curve. Shevtov et al. [6] discussed the procedure for the design of profile for the wheel that depends on the geometry of the wheel/rail connection, which utilizes a mathematical optimization modus operandi, the practice was adopted from the cluster of Railway engineers at the Delft University of Technology. It was concluded that all factors affecting the vehicle dynamics, wear, maintenance cost etc. should be considered by engineers on wheel profile control. Emblom and Berg [9] discussed the braking as well as wheel/rail connection requirement in relations to the simulation of wheel wear and the impact of disc deceleration as well as the state of lubrication and friction. It was discovered that, applying rigid creep gives a mean sliding velocity distribution underestimating the slip towards rear of the contact patch, but for a fully developed slip consisting of small spin, this discrepancy vanishes. Braghin et al. [10] discussed a predictive representation for wear in comparison with the optimum experimental test on wheelset in a laboratory. It was concluded that wheel reprofiling at every 200,000 km range is almost twice the service life of the wheel and reduces the whole life cycle cost.

Polach [11] built up an approach to wheel profile design rooted in an established spread of contact points on the wheel profile. Shevtsov et al. [12] presented a wheel profile design that strengthens the wheel/rail interface through eliminating wear, under conditions of Rolling Contact Fatigue (RCF). It was concluded that the shape and the wheel/rail profile impacts on the RCF damage. Frohling [13] reported the examination of asymmetric wheel profile and its impacts. It was concluded that the technical solution of the problem entails the buildup of a wheel and rail profile and the optimization of the grinding process. Wu [14] discussed the influences of wheel/rail profiles on vehicle curving and lateral stability. Zobory [15] presented the extended sphere of problems of wheels and rail wear prediction and the recent results revealing the current knowledge. Sapronova et al. [16] presented a number of manners of avoiding deterioration of wheels and rails. Zakharov et al. [17] described a scientific approach to profiles evolution and optimization using combined scientific and practical methods. Kalker [18] offered a technique of computing the development through wear of railway wheels. Li et al. [19] described a direct optimization technique for railway wheel profile based on the weighed nominal gap between wheel and rail at the point of contact.

Kalousek and Megel [20] discussed a pummeling model being derived at the National Research Council Centre for Surface Transportation Technology to appraise the performance of rail profiles when loaded by a large number of measured new and worn wheels. Markine et al. [21] presented a method for improved design of a wheel profile based on geometrical wheel/rail contact properties such as the Rolling Radii Difference (RRD). Wang and Masood [22] investigated the influence of draw die geometry on the sheet metal tool wear distribution over the draw die radius by applying numerical and experimental procedures. Cho et al. [23] suggested a new technique for determining the profile of an original unworn surface. Brouzoulis et al. [24] presented a method for evaluating the Archard’s wear co-efficient from data collected in a full-scale wheel-rail test rig i.e. under realistic loading conditions. Tassini et al. [25] discussed the implementation of a numerical model of the twin disc arrangement, which multiplies the distribution of tangential forces over the contact patch between two discs. Vuong et al. [26] experimentally validated and implemented a recently proposed mechanics based model for the wear co-efficient of rail steels to achieve useful predictive wear models for some conversant railway steels. Yuan et al. [27] compared various procedures to calculate profile and surface dimension. Grzesik [28] discussed an extensive characterization of the surface roughness generated during hard turning (HT) operations performed with conventional and wiper ceramic tools at different feed rate and its changes originated from tool wear. However, other areas in which research in literature on wheel/rail profile concentrates on the wheel only, wheel and rail, tool profile, mill liner profiles, die radius arc profile, railway wheel profile and are thus explained thereafter. Abdullah et al. [29] reported on wheel wear and work piece surface roughness in creep-feed grinding of tungsten carbide with 20% cobalt binder using a resin-bonded nickel-coated diamond wheel. Jahed et al. [30] presented a method for determination of optimum railway vehicle wheel profile for a given geometric contact characteristics of wheel/rail using a numerical optimization approach. Li et al. [19] considered the effects of axle load, the angle of attack, rail hardness and lubrication on wear behaviors of wheel flange and rail gauge corner.

Zakharov and Zharov [31] derived equations which made possible to find the optimal wheel flange/rail head profile which provides for minimal wear rate of wheel travelling along the selected track section, once the lateral forces and the angle of attack and the wear model are known. Chongyi et al. [32] also developed a wheel/rail profile wear prediction methodology and applied it to wheel/rail disc test about the wear of flange and gauge. Rovira et al. [33] used an experimental method based on the measurement of ultrasonic reflection to solve the contact problem together with a FASTSIM (simplified theory of rolling contact) algorithm. Braghin et al. [10] presented a fast and reliable wear prediction model that has been validated through comparison with full-scale experimental tests carried out on a single mounted wheelset under laboratory conditions. The following Table 1 offers a summary of some other parameters considered in wheel/rail studies:

2.4 Literature Summary

The proceeding review of literature reveals the fast growing attention in enhancing the wear performance of the wheel/rail interaction being promoted by the ever increasing economic activities limiting the funds available for maintenance of rail/wheel tracks for railway

Table 1: Wheel/rail wear parameters used in previous studies

S/N	Author	Parameter utilized in the paper
	Dirks and Enblom. [34]	Contact position, size of the contact area, frictional coefficient, tangential resultant creep force, vertical force, longitudinal creep force, lateral creep, surface fatigue index, value of the wear number, damage index, wear volume, sliding distance, normal force, hardness of the material, wear coefficient, contact environment compensation factors, contact patch, area of the element, contact pressure, sliding velocity, running speed, position in the contact patch, elastic contribution, curve radius, vehicle speed, contact sizes, wheel revolution, wear number, fatigue probability, yaw angle, creep forces, longitudinal tangential force, curve radius
2.	Polach [11]	Equivalent conicity, contact size, normal stress, normal parameters, contact angle, nominal clearance, nominal gauge system, lateral wheel distance in the nominal position, wheel set displacement, specified wheel rolling radius difference function stimulated coefficient
3.	Chongyi et al. [32]	Angle attack, radial load, thrust load, rolling velocity, rail cant, surface hardness of rail, test load, axle load, normal pressure and slip velocity, wear depth, length of wear step, wear step, contact pressure, contact tangential traction, contact stiffness, friction coefficient, equivalent stress, critical stress, slip velocity, equivalent slip velocity, kinetic friction coefficient, static friction coefficient, decay coefficient, equivalent slip velocity, volume of material removed from the surface, sliding distance, nominal force, hardness of the worn material, wear coefficient, nodal wear velocity in the depth direction, nodal contact area, interface normal pressure, wear coefficient of the hardened tool steel on the hardened tool steel, wear depths of the surface nodes, maximum wear depth
4.	Abdullah et al. [29]	Depth of cut, wheel width, wheel diameter, undeformed cutting chip thickness, side wheel wear, radial wheel wear, cutting speed, average grit size of the abrasives, depth of the ground groove
5.	Cho et al. [23]	Cutoff angular frequency, angular frequency at minimum attenuation, sampling interval, velocity of the stylus, cut-off wavelength, number of data points, height of measured surface profile prior to the wear test, height of the measured surface profile after the wear test
6.	Ipek and Selcuk [35]	Pressure, highest roughness surface
7.	Wang and Masood [22]	Total time of stroke, sliding velocity at time, normal contact pressure at time, contact area, contact pressure on each element of the die radius surface, the distance of the sliding movement of the blank, contact pressure distribution pattern die radius, blank holder pressure, radius of the profiles
8	Jahed et al. [30]	Rolling radius difference, wheel equivalent conicity, track angle, rail inclination angle, wheel diameter, unbalanced axle load, deviation from the target function, slope of the chord

lines. Although existing research, and particularly the work credited to Li et al. [2] tackled the wheel/rail interface problem to propose the wear development for the profile, the restriction of ignoring surface fatigue strength and design factors cannot be overlooked but must be quickly corrected for the most advantageous evaluation of the wear build-up for the wheel/rail interface. To overcome these restrictions, several scholars have provided different representations. However, these models often ignore some key elements of the Li et al. [2] model while incorporating merely the fatigue strength at the exclusion of design factors. This is unacceptable as it does not bring complete benefit of a total system's picture in evaluation and decision making process.

While acknowledging these previous studies, the outstanding development and transformation of the Li et al's [2] work is still a great expectation of the research community. It is believed, through this research that time has come to reveal this literature shortcoming and correct it with some novel ideas. The new application of dimensional analysis framework to incorporate surface fatigue strength and the design factors into the Li et al. [2] proposed model may be the framework that the community is expecting. So, the literature definition of surface fatigue strength and design factors are infused into the Li at al.'s [2] to transform it into a new structure that may be relied upon by the research community to predict the wear development for the wheel/rail profile. Consequently, the newly developed model provides a new structure with which future models can be compared, and therefore offers a research answer to the sub-optimal platforms in existing literature.

3. Methodology

This section discusses how the Li et al.'s [2] Archard's wear model was modified as applied to wheel/rail profile. It also discusses the calculation of the wear volume in a damped and undamped situation. The wheel is modeled as moving on a curved track, whereas, the vehicle is of 8 wheels classified into the front and rear bogie. The concern in modeling and analysis include the area of contact, the contact pressure, property of the wheel/rail surface, mode of contact, size of contact property of the material producing wear, the wear volume and the time the material was removed from the surface.

3.1 Application of Dimensional Analysis to Wear Profile Problem Formulation

Archard's wear model for sliding contact is considered in calculating the wear locally. The wear model stated that, the material volume loss due to wear depends on, contact force, sliding distance and material hardness, as described by Li et al. [2]:

$$V_{wear} = K\frac{PS}{H} \tag{1}$$

where V_{wear} is the wear volume, P is the normal force, S is the sliding distance, H is the hardness of the worn material and K is the wear co-efficient. However, F is taken from the vehicle modeled as a multi-body system with 35 degrees of freedom.

Since pressure, $P = F/A$ (Norton, 2006), thus

$$V_{wear} = K\frac{F_{35}S}{A_r H} \tag{2}$$

where A_r is the real area of contact and is affected by the asperities present on the surfaces of the mating parts. The tops of the asperities will be in critical contact with the mating part and the initial area of contact will be extremely small. The resulting stresses in the compressive yield strength of the material. As the mating force is increased, the asperity tips will yield and spread until their combined area is sufficient to reduce the average stress to a sustainable level i.e. some "compressive penetration strength" of the weaker material (Norton [8]). However, the real area of contact (Norton [8]), can then be estimated from;

$$Ar \cong \frac{F}{Sp} \cong \frac{F}{3\,Syc} \tag{3}$$

By considering the friction on the wheel profile in relation to the real area of contact, it refers to the coulombs sliding friction (Norton [8]):

$$f = \mu\, F \tag{4}$$

Norton [8] however states that, the normal force presses the two surfaces together and creates an elastic deformations and adhesion at the asperities' tips. Thus, define coulombs friction force *f* as being the force necessary to shear the adhered and elastically interlocked asperities in order to allow a sliding motion, this shearing force is equal to the product of the shear strength of the weaker material and the actual contact area A_r, plus a "plough force" P.

Therefore (Norton [8])

$$f = S_{us} A_r + P \tag{5}$$

Notice that *P* is due to lose particles digging into the surfaces and can be neglected compared to the shear force. Therefore, using real contact area as expressed Norton [8] we have;

$$Ar \cong \frac{F}{3Syc} \tag{6}$$

Recall $f = S_{us}\ Ar$ (ignoring P)

From Norton [8], therefore we have

$$Ar = \frac{f}{S_{us}} = \frac{F}{3Syc} \quad = \frac{f}{F} = \frac{S_{us}}{3Syc} = \mu \tag{7}$$

Substituting Equation (7) into Equation (2) gives

$$V_{wear} = K\,\frac{F_{35} S_j}{\mu H} \tag{8}$$

where V_{wear} is the volume of wear, *K* is the wear co-efficient, *S* is the sliding distance, *H* is the hardness and μ is the co-efficient of friction and F_{35} is the force at the 35th degree of freedom.

$$V_{wear} = k\,\frac{\sigma_{35} S_j A}{\mu H} \tag{9}$$

Furthermore, Li et al. [2] calculated the sliding distance of the wheel as,

$$S_j = \left| Vslip(j) \right| \frac{\Delta x}{vo} \tag{10}$$

Where v_0 is the vehicle speed, Δx is the wheel rolling distance pr step, V_{slip} velocity component between the wheel and rail, and (Li et al. [2]):

$$V_{slip}\ (j) = v_0 \begin{bmatrix} \xi_1 - Y_j \xi_3 - \dfrac{\partial u_1}{\partial x} \\ \xi_2 - X_j \xi_3 - \dfrac{\partial u_1}{\partial y} \end{bmatrix} \tag{11}$$

where QUOTE $U_{ij} U_{ij}$ (*i*= 1, 2) are the components of the elastic displacement difference at the center of element *S* in the direction of *x* and *y, x_j* and y_j are the coordinates of *x* and *y*, respectively, at the center of element *j*, it is obvious that S_j at the different element in the contact area is different.

3.2 Wear Volume by Considering Surface Fatigue Strength and the Design Factor of the Wheel/Rail Profile

The problem of two machine elements, i.e. the wheel-rail element mating with one another by rolling, sliding or a combination of rolling and sliding contact or a combination of rolling and sliding contact this properly of mating materials is called surface endurance shear. Using Buckingham model on fatigue, where he defined a load stress factor also called a wear factor which is derived from the Hertz equation as described by Shigley and Mischke [7] where;

$$b = \sqrt{\frac{2F}{\pi l}\,\frac{\left(1-v_1^2\right)/E_1 + \left(1-v_2^2\right)/E_2}{\left(1/d_1\right) + \left(1/d_2\right)}} \tag{12}$$

$P_{\max} = \frac{2F}{\pi bl}$; Where L is change to width, and the Diameter is change to r, let $2r=d$; thus, we remove the square root we have the expression given by Shigley and Mischke [7].

$$b^2 = \frac{4F}{\pi w} \frac{\left(1-v_1^2\right)/E_1 + \left(1-v_2^2\right)E_2}{1/r_1 + 1/r_2} \tag{13}$$

Then since Shigley and Mischke [7] expressed that;

$$P_{\max} = \frac{2E}{\pi bw}, \tag{14}$$

$$then\ Sc = \frac{2F}{\pi bw} \tag{15}$$

Where S_c is called, the contact strength or surface fatigue strength.

More so, relating the allowable surface stress σ_c and the design factor. (Shigley and Mischke [7]) stated that;

$$\sigma_c = c_p \sqrt{\frac{F_{35}}{W_{nd}}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)} = \frac{C_p}{\sqrt{n_d}} \sqrt{\frac{F}{w}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)} = \frac{Sc}{\sqrt{nd}} \tag{16}$$

But since the equation is focused on stress, $\sigma_c = Sc/nd$. Thus, using this equation in Equation (9) we would have an equation that predicts the amount of wear to be expected when designing a wheel wear profile and/or considering the wear depth on a wheel/wear profile. Thus, we have.

$$V_{wear} = \frac{KS_c\ S_j A}{\mu H n_d} \tag{17}$$

Therefore

$$V_{wear} = \frac{KS_c S_j A}{\mu H n_d} \tag{18}$$

3.3 Wear Volume in a Damped and Undamped Situation

In calculating the wear volume for the Rail vehicle in vibration, it is required that we consider the factors that causes wear when two surfaces are pressed together under load and vibration, however the following parameters were considered in calculating for the wear volume of the wheels in a damped and undamped situation; a damped and undamped vibration, the modified Li et al.'s [2] model of Archard's model, the kinematic viscosity of the lubricant to be used, and the Buckingham's model.

3.3.1 Wear volume of a damped body under vibration

The equation of a damped body under vibration as defined as (Desilva [36]):

$$W_d = \sqrt{\frac{k_0}{m} - \left(\frac{d}{4m}\right)^2} \tag{19}$$

However, we modify the Equations for the vibration of a damped body by using the Equation for natural frequency of a damped and undamped vibration expressed by Desilva [36]:

$$W_d = 2\pi f \tag{20}$$

$$W_n = \sqrt{\frac{k_0}{m}} \tag{21}$$

Then we say that;

$$2\pi f = W_n - \frac{d}{4m} \tag{22}$$

However, as described by Desilva [36], the critical damping is calculated as $C_c = 2mW_n$ thus, Equation (22), $8\pi mf = 2C_c - d$ (23)

More so, the critical damping can be calculated as $C_c = \frac{k_0}{\pi f}$ (Desilva [36]), therefore,

$$2C_c = 2\left(\frac{k_0}{\pi f}\right) = 8m\pi f + d \tag{24}$$

$$C_c = 2k_0 = 8m\pi^2 f^2 + 2d\pi f \tag{25}$$

$$m = \frac{k_0 - d\pi f}{4\pi^2 f^2} \tag{26}$$

Furthermore, considering the kinematic viscosity of the lubricant being used, kinematic viscosity of a fluid is defined as the ratio of the dynamic viscosity of a fluid to the density, thus explained below:

$$v = \mu / \rho \tag{27}$$

Also, recall that density as expressed by Kumar [37] is:

$$\rho = m / v \tag{28}$$

Thus, $m = \frac{\mu V}{v}$ (29)

From Equations (26) and (29), we have

$$m = \frac{(k_0 - d\pi f)}{4\pi^2 f^2} \tag{30}$$

Therefore, substituting Equation (30) in Equation (18), we have;

$$V_{wear} = \frac{KS_c S_j \pi A}{\mu Hnd}$$

But however, recall that in Norton [8], it was given that

$$A = \frac{F}{3S_{yc}} \tag{31}$$

$$F = ma \tag{32}$$

Thus,

$$A = \frac{ma}{3S_{yc}} \tag{33}$$

Therefore,

$$V_{wear} = \frac{kS_c S_j ma}{\mu Hn_d 3S_{yc}} \tag{34}$$

Substituting Equation (30) in Equation (34)

$$V_{wear} = \frac{kS_c S_j a(K_0 - d\pi f)}{12\mu Hn_d S_{yc}\pi^2 f^2} \tag{35}$$

Equation (20) defines the wear volume for a damped situation of vibration of the wheel/rail profile.

3.3.2 Wear volume of an undamped body under vibration

However for an undamped body, the Equation of an undamped body under vibration as defined as (Desilva [36]):

$$W_n = \sqrt{\frac{k_0}{m}} \tag{36}$$

However, we modify the Equations for the vibration of an undamped body by using the Equation for natural frequency of an undamped vibration as explained below.

$$2\pi f = \sqrt{\frac{k_0}{m}} \tag{37}$$

Substituting for m from Equation (37) into Equation (18) and noting Equation (33) gives

$$V_{wear} = \frac{kS_c S_j k_o a}{12\mu Hn_d S_{yc}\pi^2 f^2} \tag{38}$$

Equation (38) defines the wear volume for an undamped situation of vibration of the wheel/rail profile.

3.4 Prediction of Wear Volume

However, linear regression analysis will be used to develop a statistical model to make predictions. The reserach process is presented in Figure 1. Regression analysis is used to find Equations that fit data. Once we have the equation, we can use the statistical model to make prediction for wheel rail profile evolution due to wear, and also it will be used to predict the wear volume of the wheels in a damped and undamped situation. However, from elementary algebra, the Equation for a line as defined by Stroud [36] is $y = mx + b$. This approach is used to calculate linear regression using simulated data, and thus find the Equation $y = a + bx$.

4. Results and Discussion

Eight different wheelsets have been considered in this study, by modifying Archard's wear model using surface fatigue strength and design factor as defined by Buckingham's model, and also, using linear regression analysis to derive a predictive model to determine the wear volume, V_{wear}, to be expected when the wheels moves along the curved track at sliding distance S_j, the wheels are classified as the front and rear bogie, the front bogie contains wheel 1, 2, 3 and 4, while the rear bogie contains 5, 6, 7 and 8, however these wheels have been grouped by combining the leading wheels (i.e. wheel 1, 2, 5 and 6) as Wheel I and the trailing wheels (wheel 3, 4, 7 and 8) as Wheel II. The results for the wear volume for the current study was obtained from simulated data and compared with the wear volume obtained for Li et al. [2]. The comparison for the total wear volume is shown in Figure 2. It was observed that wheel 1 has the greatest wear volume with a total wear volume of 0.33238 mm^2 and wheel 4, has the least wear volume of 0.036486 mm^2.

This result when compared with the results obtained from Li et al. [2] work, shows the same results for these two wheels having the highest and the least wear volume.

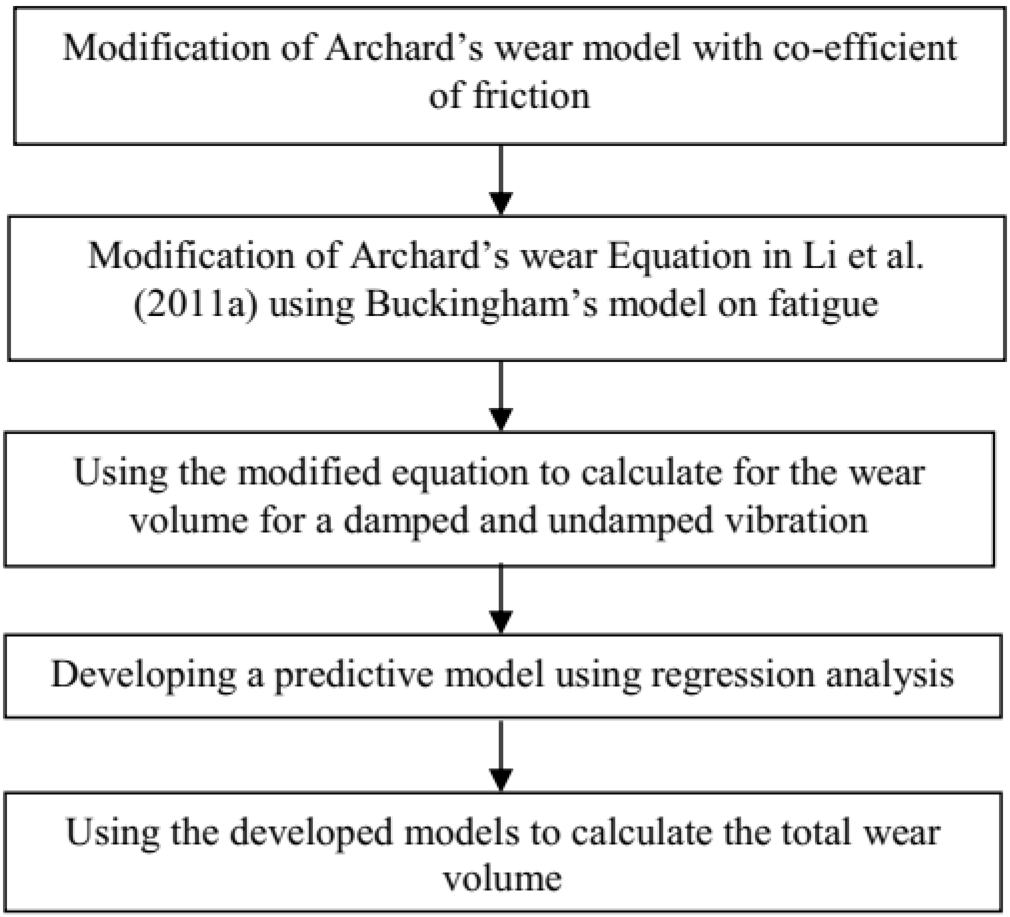

Figure 1: Research Process

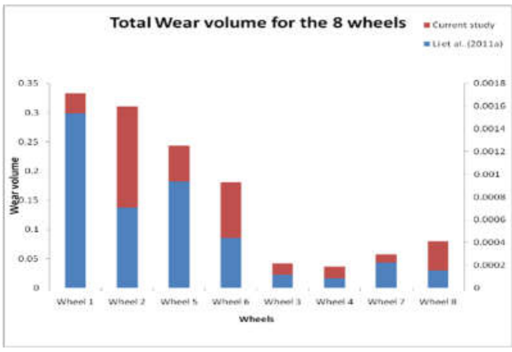

Figure 2: Total wear volume of the 8 wheels

However, the wear volume is obtained when parameters such as surface fatigue wear and design factor are considered. Furthermore, the results obtained shows that the stress at the surface of the wheel has a great effect on the volume of material to be removed from the surface of the wheel. Therefore, the strength of material to be used for constructing the wheel should be high enough to resist easy wear at the surface. Table 2-3 show the comparison of the result obtained by Li et al. [2] and the current study.

Furthermore, through linear regression analysis a predictive model has been formed, and

Table 2: Wear volume of wheels 1, 2, 3 and 4 for Li et al. [2] and current study

	Wheel 1			Wheel 2			Wheel 3			Wheel 4		
		Wear volume for wheel 1			Wear volume for wheel 2			Wear volume for wheel 3			Wear volume for wheel 1	
x	S_j	Li et al.	Current result	S_j	Li et al.	Current result	S_j	Li et al.	Current result	S_j	Li et al.	Current result
0	0.6	0.000045	0.0143	0.6	0.000045	0.0143	0	0	0	0	0	0
80	0.2	0.0000187	0.00475	0.1	0.00000649	0.00477	0.02	0.0000017	0.000477	0.02	0.0000013	0.000477
160	1.2	0.0000131	0.0285	1	0.0000527	0.0239	0.08	0.00000747	0.0019	0.08	0.00000438	0.00191
240	1.4	0.000159	0.0335	1.2	0.0000487	0.0286	0.16	0.0000162	0.0143	0.26	0.0000127	0.0062
320	1.2	0.000127	0.0285	1.15	0.0000513	0.0274	0.17	0.0000166	0.00405	0.18	0.00000406	0.0019
400	0.8	0.0000747	0.0119	1.2	0.0000779	0.0286	0.04	0.00000357	0.000958	0.08	0.00000487	0.0019
480	0.2	0.0000154	0.00477	0	0	0	0.02	0.00000166	0.000477	0.04	0.00000268	0.000954
560	0	0	0	0.2	0.000015	0.00477	0.02	0.00000146	0.000477	0.02	0.0000015	0.000477
640	1.15	0.000117	0.0214	0.9	0.0000456	0.0214	0.12	0.0000112	0.00287	0.14	0.00000767	0.00334
720	1.2	0.000117	0.0286	0.9	0.0000493	0.0215	0.1	0.00000892	0.0024	0.14	0.00000795	0.00334
800	1.2	0.00012	0.0286	0.9	0.0000475	0.0215	0.11	0.00001	0.00262	0.15	0.00000821	0.00358
880	0.3	0.000268	0.00719	1	0.0000609	0.0238	0.08	0.00000698	0.0019	0.06	0.00000353	0.00143
960	0.05	0.00000365	0.00119	0	0	0	0.04	0.00000284	0.000954	0.04	0.000003	0.000954
1040	1.2	0.0000974	0.0286	1.7	0.0000412	0.0167	0.02	0.00000178	0.000479	0.07	0.00000426	0.00167
1120	0.9	0.0000858	0.0214	0.7	0.0000369	0.0167	0.06	0.00000536	0.00143	0.08	0.00000438	0.00191
1200	0.9	0.000084	0.0215	0.7	0.0000383	0.0167	0.08	0.00000714	0.00192	0.07	0.00000398	0.00167
1280	0.9	0.0000858	0.0214	0.7	0.0000369	0.0167	0.08	0.0000073	0.0019	0.08	0.00000438	0.00191
1360	0.9	0.0000858	0.0215	0.75	0.0000426	0.0179	0.08	0.00000714	0.00192	0.08	0.00000438	0.00191
1440	0.2	0.0000187	0.00478	0.2	0.0000118	0.00476	0.02	0.0000017	0.000475	0.04	0.00000227	0.000954
1520	0	0	0	0	0	0	0	0	0	0	0	0
1600	0	0	0	0	0	0	0	0	0	0	0	0

Table 3: Wear volume of wheels 5, 6, 7 and 8 for Li et al. [2] and current study

	Wheel 5			Wheel 6			Wheel 7			Wheel 8		
		Wear volume for wheel 5			Wear volume for wheel 6			Wear volume for wheel 7			Wear volume for wheel 8	
x	S_j	Li et al.	Current result	S_j	Li et al.	Current result	S_j	Li et al.	Current result	S_j	Li et al.	Current result
0	0.6	0.000045	0.0143	0.6	0.000045	0.0143	0	0	0	0	0	0
80	0.1	0.00000892	0.0024	0.1	0.0000071	0.00239	0.02	0.00000179	0.000479	0.02	0.0000013	0.000477
160	0.8	0.0000747	0.019	0.4	0.0000235	0.00952	0.14	0.0000139	0.00333	0.14	0.0000071	0.00333
240	0.9	0.0000931	0.0215	0.7	0.0000312	0.0167	0.28	0.0000301	0.00668	0.42	0.0000179	0.01
320	0.8	0.0000811	0.019	0.75	0.0000335	0.0179	0.24	0.0000243	0.00571	0	0.0000214	0.0105
400	0.7	0.0000653	0.0167	0.6	0.0000389	0.0119	0.16	0.000014	0.00381	0.16	0.00000974	0.00381
480	0	0	0	0	0	0	0.02	0.000017	0.000475	0.02	0.00000133	0.00525
560	0	0	0	0	0	0	0.02	0.00000146	0.000477	0.02	0.0000015	0.00477
640	0.6	0.0000584	0.0143	0.5	0.0000304	0.0119	0.28	0.0000278	0.00665	0.26	0.0000132	0.00619
720	1	0.0000913	0.0238	0.6	0.0000329	0.0143	0.2	0.0000187	0.00478	0.24	0.0000131	0.00571
800	1	0.0000933	0.0238	0.6	0.0000316	0.0143	0.19	0.0000185	0.00453	0.24	0.0000127	0.00571
880	0.6	0.0000536	0.0144	0.55	0.0000335	0.0131	0.18	0.0000168	0.0043	0.11	0.00000647	0.00262
960	0	0	0	0	0	0	0.01	0.00000071	0.00239	0.01	0.000000751	0.000239
1040	0.6	0.0000481	0.0143	0.1	0.00000649	0.00239	0.01	0.000000893	0.00024	0.01	0.000000609	0.000238
1120	0.6	0.0000548	0.0143	0.6	0.0000341	0.0143	0.14	0.0000131	0.00335	0.14	0.00000738	0.00334
1200	0.6	0.0000535	0.0144	0.5	0.0000294	0.0119	0.16	0.0000143	0.00382	0.18	0.00000986	0.00429
1280	0.6	0.0000548	0.0143	0.5	0.0000284	0.0119	0.16	0.0000149	0.00382	0.19	0.0000102	0.00453
1360	0.6	0.0000535	0.0144	0.5	0.0000289	0.0119	0.04	0.00000365	0.000952	0.19	0.00001	0.00453
1440	0.1	0.00000933	0.00239	0.1	0.00000588	0.00238	0.08	0.00000698	0.00191	0.18	0.0000102	0.00429
1520	0	0	0	0	0	0	0	0	0	0	0	0
1600	0	0	0	0	0	0	0	0	0	0	0	0

an intercept of 0.0000185 and an X-variable of 0.023466 were obtained as the coefficients, thus yields the Equation (39):

$$Y = 0.0000185 + 0.023466X \quad (39)$$

For wheel 1, which is a member of the Wheel I, it is observed from calculations that, the wear volume is more compared to the wear volume obtained for wheels' 2, 3, 4 in the front and wheels' 5, 6, 7, 8 in the rear bogie. This variation in the result is as a result of more pressure that is exerted on the wheel when moving on the curved track. Moreover, more work is done by the wheel in changing geometry and the direction of movements as compared to the other wheels. Although the wear volume of wheel 2 is high, it is however lower when compared with what is obtained for wheel 1. The total wear volume for wheel 2 is high due to the fact that, wheel 2, is a member of the leading wheelset in the front bogie, thus, more pressure and work is done, but lesser pressure is exerted, and lesser work is done as compared with wheel 1

Wheel 5, is a member of wheel I but located in the rear bogie, it is observed from the result and graph obtained that, as a leading wheelset in the rear bogie, it does more work in changing geometry and direction of movement on the curved track when compared to the other wheels in the rear bogie and also more pressure is exerted on this wheel. This has resulted in the total wear volume of 0.24329 mm^2 as obtained in the simulated data. However, one can as well say that, wheel 5 follows the same principle of operation as wheel 1, but with lesser work performed on the curved track. Wheel 6 is located at the rear bogie, but being a member of the leading wheelsets (i.e. wheel 1), also has a higher wear volume compared to Li et al. [2] as observed on the graph. The result obtained for wheel 6, shows it has the least wear volume in the leading wheelset of the vehicle. Wheel 3 being a trailing wheel for the front bogie has a much lower wear volume, with a total wear volume of 0.041507 mm^2. This is as a result of lower pressure exerted and the lesser work is performed by this wheel. Thus, stress at this point is reduced compared to leading wheelset 1 and 2. Furthermore, wheel 4 has the least wear volume of 0.036486 mm^2 when considering the front bogie and rear bogie. This shows that the region of the lowest concentration is on this wheel. Other wheels such as wheel 7 and 8 have the lowest wear volume in the rear bogie as observed. Thus, this indicates lower stress and work done on these trailing wheels when compared to the leading wheels 5 and 6 in the same rear bogie. However, from the results obtained from the model in the research work, it can be deduced that, more pressure is exerted on wheels' in group wheel I, and, more work is performed by wheels' in group wheel I in changing geometry and direction of movement on the curved rail. Furthermore, the leading wheelsets in each bogie (i.e. front and rear) have a high wear volume compared to the trailing wheelset. Figure 2 shows the total wear volume of the wheels, with wheel 1 having the highest total wear volume and wheel 4 showing the least wear volume.

For the wear volume when the wheels are damped and undamped, Eight different wheel were also considered in this study, by modifying Archard's wear model using surface fatigue strength and design factor as defined by Buckingham's model and the equation for a damped and undamped vibration. Furthermore, linear regression analysis is used to derive a predictive model to determine the wear volume, V_{wear}, to be expected when the wheels move along the curved track at sliding distance S_j.

The results for the wear volume for the damped and undamped vibration of the wheels were obtained from simulated data and compared, the result is shown in Table 4(a)-(h) and the comparison is shown in Figure 11. It was observed that wheel 1 has the greatest wear volume with a total wear volume of 0.0003928 mm^2 for damped vibration and a total wear volume of 0.00034457 mm^2 for undamped vibration and wheel 3, has the least wear volume of 0.000035248 mm^2 for damped and a total wear volume of 0.000030808 mm^2 for undamped vibration. However, the wear volume is obtained when parameters such as surface fatigue wear and design factor in relations to damped and undamped vibrations are considered. Furthermore, the results obtained shows that the frequency of vibration, the pressure and the stress at the surface of the wheel has a great effect on the volume of material to be removed from the surface of the wheel.

Using linear regression analysis, a predictive model has been formed, and an intercept of 0.0000000000659 and an X-variable of 0.000000271 were obtained as the coefficients for the damped vibration and an intercept of -

0.000000023 and an X-variable of 0.0000238 were obtained as the coefficients for the undamped vibration. Thus, Equation (40) and (41) are used to predict the wear volume for damped and undamped vibration respectively;

$$Y = 0.0000000000659 + 0.000000271X \qquad (40)$$

$$Y = -0.000000023 + 0.0000238X \qquad (41)$$

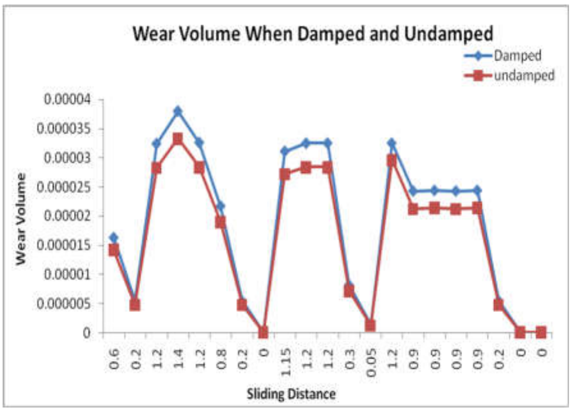

Figure 3: Wear volume for wheel 1 when damped and undamped

It was observed that the wear volume for wheel 1 is high. This is due to the fact that more pressure is being exerted on this wheel as it moves on the curved track. More so, wheel 1 is a member of the leading wheel set pulling the other wheels, thus more work is done. Furthermore, it has also been observed that the wear volume for a damped vibration is more when compared to the wear volume of an undamped vibration. Therefore, this explains that when the wheels are being damped more wear volume should be expected as compared to a situation of lower degrees of damping.

Wheel 2 also shows a high wear volume being a member of the leading wheel set. This can also be attributed to higher work done by the leading wheelsets. However, less pressure is exerted on this wheel when compared to wheel 1. The wear volume of a damped vibration is more compared to an undamped vibration, thus fatigue in undamped vibration is less as compared to a damped vibration.

Wheel 5 being a member of the leading wheelset for the rear bogie shows a lower wear volume as compared to wheel 1 and 2. However, the wear volume on this wheel is more than what was obtained in the rear wheelset's 7 and 8.

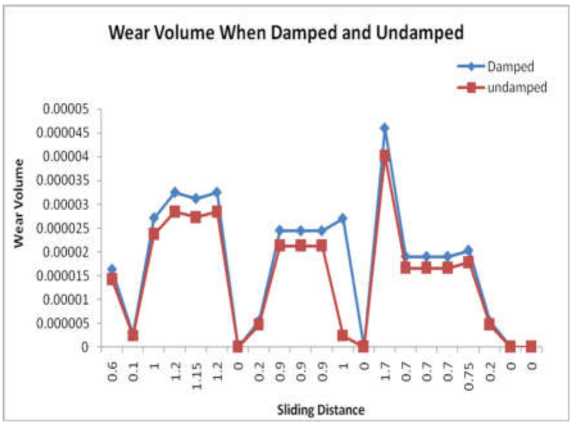

Figure 4: Wear volume of wheel 2, when damped and undamped

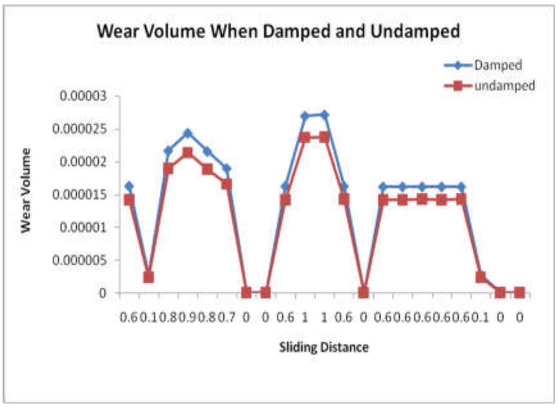

Figure 5: Wear volume of wheel 5, when damped and undamped

This is as a result of more work being performed as the leading wheel in the rear bogie. Thus, more work is done. More so, the pressure at the wheel is more when compared to the other wheels at the rear bogie, although the wear volume is not the same as that of wheel 1. The wear volume for a damped body is more than the undamped body due to vibration.

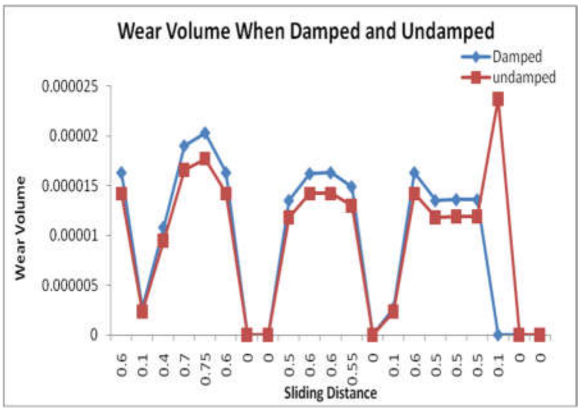

Figure 6: Wear volume of wheel 6, when damped and undamped

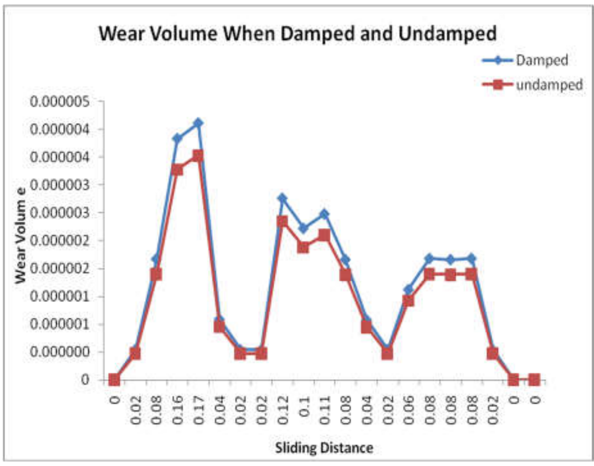

Figure 7: Wear volume of wheel 3, when damped and undamped

It was observed that wheel 3 has the least total wear volume. This is attributed to lesser worked done by this wheel as being a member of the trailing wheelset in the front bogie and, lesser pressure is being exerted on this wheel while moving along the curved track. Thus, the lesser a damped vibration as more wear volume when compared with an undamped vibration

Wheel 4 also has a low wear volume of 0.000038632 mm^2 as a member of the rear bogie. This shows that the wheel is also in a region of the low concentration of stress and pressure. Also, lesser work is done by this wheel in moving on the curved track.

Other wheels, such as wheel 7 and 8 have lowest wear volume for damped and undamped vibration for the rear bogie. Thus, this indicates lower pressure, stress and work done at the trailing wheels of the rear bogie. However, it is observed that wheels 7 and 8 have more wear

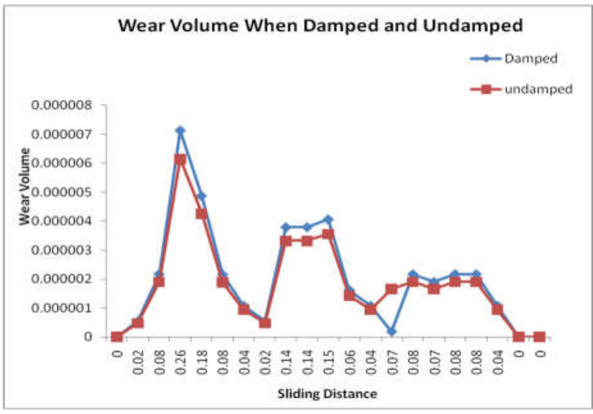

Figure 8: Wear volume of wheel 4, when damped and undamped

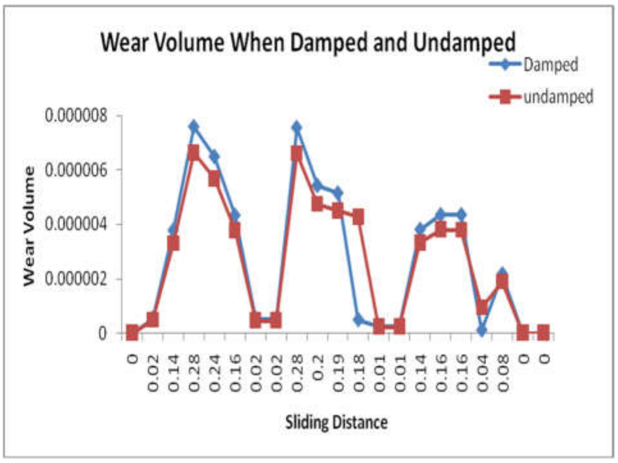

Figure 9: Wear volume of wheel 7, when damped and undamped

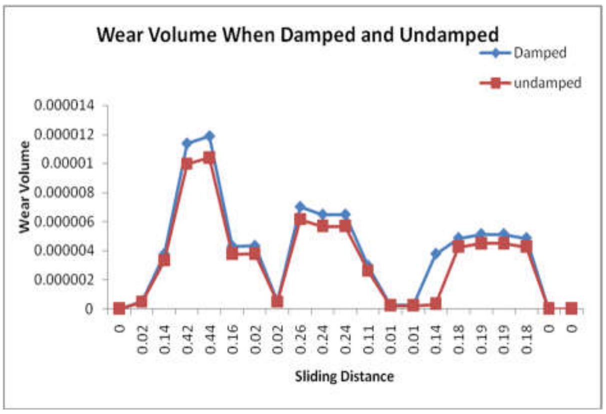

Figure 10: Wear volume of wheel 8, when damped and undamped

volume when compared with wheel 3 and 4. This is due to more work, pressure and stress concentrated at wheel 7 and 8, compared to wheel 3 and 4. Figure 11 shows the graph representing the comparison between the wear volumes obtained for damped and undamped vibrations. It shows the comparison between the total wear volume for the Damped and the Undamped vibration on each of the 8 wheels moving on the curved tracks. Table 4 shows the results obtained for the wear volume.

From Table 4, it is concluded that the total wear in a damped vibration for each of the wheels is more than the corresponding total wear volume for the undamped vibration for each of the wheels. This however explains that, more wear volume should be expected from a damped vibration compared to an undamped vibration. This is attributed to the increased wheel/rail contact in a damped situation compared to an undamped vibration.

Table 4: Total wear volume for damped and undamped vibrations in the wheels

Wheels	Damped	Undamped
Wheel 1	0.0003928	0.00034457
Wheel 2	0.00036044	0.00030791
Wheel 5	0.00027624	0.00024206
Wheel 6	0.000206047	0.0002036
Wheel 3	0.000035248	0.000030808
Wheel 4	0.000042513	0.000038632
Wheel 7	0.000052889	0.000055223
Wheel 8	0.000084235	0.000070684

However, from the results obtained from the model for the damped and undamped vibration of the wheel in this research work, it can also be deduced that, more work is performed by the wheelset in Wheel I in changing geometry and direction of movement on the curved rail. This has produced high wear volume in the wheels in wheel I, compared to wheel II. Furthermore, it can also be deduced that damping of the wheels, also produces larger wear volume, compared to when the wheels are with lesser degrees of freedom or undamped. Figure 11 shows the total wear volume of the wheels, with wheel 1 having the highest total wear volume and wheel 3 having the least wear volume

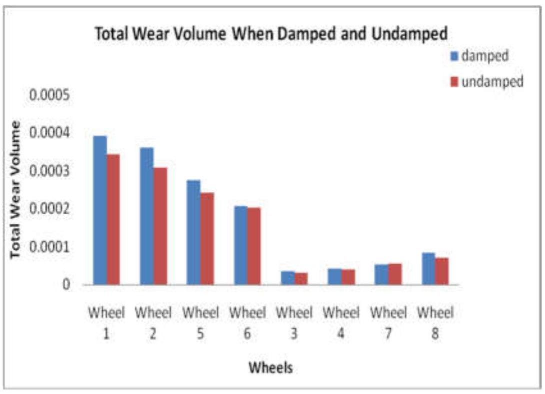

Figure 11: Comparison for total wear volume for damped and undamped vibrations in the wheels

5. Conclusions

This research work was carried out by modifying the prediction model on wheel profile evolution as proposed by Li et al. [2]. It is observed that the wheel wear volume is very small per contact area on the different coordinates of a wheel. Substantial literature and models on wheel/rail profile evolution due to wear, were studied and the results were compared. Thus, a new model was developed from the existing Archard's model as used by Li et al. [2]. This model result was compared with the current model, to validate the practicality of the current model. The current model was obtained by introducing Buckingham's model on fatigue. The model was tested and validated using data obtained from literature. The numerical result when compared with Li et al. [2] are reasonable and close, and thus, modifies Li et al.'s [2] Archard's wear model, thus improved by including surface fatigue strength and design factor parameters.

More so, the model was expanded to calculate the wear volume in a damped and undamped vibration. It was observed that, the wear volume to be expected for the damped vibration is more than that of an undamped vibration. Thus, the model is again modified to calculate the wear volume for a damped and undamped vibration. The surface fatigue strength of the wheels has a great impact on the reaction of the wheels to wear, as seen in the result for the current study. It is observed that, the surface fatigue strength increased the wear volume, such that, at a high surface fatigue strength, the wear volume will increase and vice versa. The contact stress at the contact area also plays an important role in the evaluation of the wheel profile evolution. It was discovered that, at high contact stress, the wear volume is high, thus, reducing the efficiency of the wheels. Moreover, it was discovered from the results in Table 2-4 that, hardness of the wheels has a significant role to play in the reaction of the wheel to wear. It is however advised that, in the design of a wheel for a rail vehicle, a consideration should be made on the thickness and/or hardness of the material to be used.

By damping, the results obtained in Table 4 and the comparison obtained in Figure 11 shows that, a damped medium produces higher wear volume compared to an undamped body when considering the surface fatigue strength and design factor in the modified Li et al.'s [2] model of Archard's model on wear volume. This is due to prolonged contact between the wheel and the rail during the movement of the wheel on the curved track. The undamped medium on the other hand has a lower wear volume. This is said to be associated with the fact that, lesser contact is established by the movement of the wheel when in contact with the rail in an undamped situation. This situation is synonymous with the case of when the wheel is damped in a lesser degree of freedom.

Furthermore, the high wear volume of the leading wheelset compared to the trailing wheelset in the front and rear bogie is attributed to the fact that, more work is done by the leading wheelset in pulling the other wheelsets. In this research work, the rail vehicle that was studied has a moving on a curved track. These however has led to wheel 1 and wheel 5, having higher wear volume compared to the other wheels (i.e. wheel 2,6,3,4,7, and 8) because; more pressure is exerted on these wheels in changing geometry and in moving along the curved track. Due to the pressure exerted on these wheels more contact stress is also applied on these wheels in changing geometry. More so, wheel 3 and 7 have the least wear volume in the trailing wheels for the front and rear bogies. This is said to be attributed to lower pressure exerted in changing geometry. It can also be said that lesser work is done by these wheels compared to the other wheels moving along these same curved tracks.

5.1 Practical Implications

The developed framework showcases some practical implications. In practice, especially in developing countries, sudden failures of wheel/rail interface are common. The results are huge losses of freight goods and humans. To add, the goodwill of the railway company would certainly prefer to avoid this. This framework can be able to assist railway companies that have had such unexpected experience to avoid future calamities and predict the possible profile of the wear concerning the wheel/rail interface of the railway line. Such a success can be attained through a pre-planned preventive maintenance activity that carries out fatigue test at the predetermined periods in the lifespan of the wheel/rail interface. This effort at micro-structural analysis can as well help in re-focusing on material innovative alternatives to replace existing materials used for wheel/rail structures. Furthermore, such a drive can assist the railway organization to plan on budgetary issues, for project expansion.

The following contribution to knowledge has been made in the current study:

- A mathematical model to predict the wheel profile evolution due to wear has been developed, which incorporates parameters such has surface fatigue strength and design factor in the existing Archard's wear model as used by Li et al. [2].

Concerning the future directions, the following should be noted:

- The wheel wear profile should be modeled for higher degrees of freedom to see the effect of surface fatigue strength and design factor. More so, it should be modeled for when the wheel is damped and undamped.
- This research has only been modeled for a wheel moving on a curved track. However, it is recommended that, future research on these works should be performed when the rail vehicle is moving on a sloppy terrain and/or a curved slope.
- For future research, it is also advised that, this research work should be performed using more than 8 wheels to see the effect of wear on these wheels. Moreover, more parameters should be incorporated into the model, thus, further modifying Archard's original model to calculate wear volume.

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APPENDIX

Equation of motion that defines the vibration from 1-35 degrees of freedom were calculated but only 4 and 5 degrees of freedom are shown for illustrative purposes.

4th Degree of Freedom

$$m_1\ddot{x}_1 + k_1x_1 + k_2x_1 - k_2x_2 + c_1\dot{x}_1 + c_2\dot{x}_2 = f_1(t) \quad \text{(A1)}$$

$$m_2\ddot{x}_2 - k_2x_1 + k_2x_2 + k_3x_2 - k_3x_3 + c_2\dot{x}_1 + c_2\dot{x}_2 + c_3x_2 - c_3\dot{x}_3 = f_2(t) \quad \text{(A2)}$$

$$m_3\ddot{x}_3 - k_3x_2 + k_3x_3 + k_4x_3 - k_4x_4 - c_3\dot{x}_2 + c_3\dot{x}_3 + c_4\dot{x}_3 - c_4\dot{x}_4 = f_3(t) \quad \text{(A3)}$$

$$m_4\ddot{x}_4 - k_4x_3 + k_4x_4 - k_5x_4 - c_4\dot{x}_3 + c_4\dot{x}_4 - c_5\dot{x}_5 = f_4(t) \quad \text{(A4)}$$

5th Degree of Freedom

Since, $m\ddot{x} + c\dot{x} + kx = f(t)$

$$m_1\ddot{x}_1 + \left(k_1x_1 + k_2x_2\right) - k_2x_2 + \left(c_1\dot{x}_1 + c_2\dot{x}_1\right) - c_2\dot{x}_2 = f_1(t) \quad \text{(A5)}$$

$$m_2\ddot{x}_2 - k_2x_1 + \left(k_2x_2 + k_3x_2\right) - k_3x_3 - c_2\dot{x}_1 + \left(c_2\dot{x}_2 + c_3\dot{x}_2\right) - c_3\dot{x}_3 = f_2(t) \quad \text{(A6)}$$

$$m_3\ddot{x}_3 - k_3x_2 + \left(k_3x_3 + k_4x_3\right) - k_4x_4 - c_3\dot{x}_2 + \left(c_3\dot{x}_3 + c_4\dot{x}_3\right) - c_3\dot{x}_4 = f_3(t) \quad \text{(A7)}$$

$$m_4\ddot{x}_4 - k_4x_3 + \left(k_4x_4 + k_5x_4\right) - \left(k_5x_5 + k_6x_5\right) - c_5\dot{x}_3 + \left(c_4\dot{x}_4 + c_5\dot{x}_4\right)\left(c_5\dot{x}_5 + c_6\dot{x}_5\right) = f_4(t) \quad \text{(A8)}$$

$$m_5\ddot{x}_4 - k_5x_4 + \left(k_5x_5 + k_6x_5\right) - c_5\dot{x}_4 + \left(c_5\dot{x}_5 + c_6\dot{x}_5\right) = f_5(t) \quad \text{(A9)}$$

The details for the rest i.e. 6th of Degree of Freedom to 35th Degree of Freedom follow the same pattern but are excluded here.