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Investigating the Behavior of Railway Sleeper for Different Cases and Track Conditions by Using Analytical and Numerical Methods

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A B S T R A C T

Railway Sleepers are one of the most important elements in the track superstructure. Their tasks are keeping the track geometry, resisting against vertical and lateral train loads and transferring the train loads to the ballast layers. In this research, the railway sleeper as a beam on the elastic foundation is modeled with finite length. In this regard, two cases including a) beam with constant section and without weight b) beam with variable section and with considering weight are considered and simulated. The results indicate that the beam deflection increases 18.42 percent for the second case in comparison to the first case. Bending moment for the second case decreases 42 percent in the middle of sleeper and it increases 12.77 percent in the rail seat in comparison to the first case. Also in this paper, the effects of ballast coefficients according to the different track conditions are investigated. By increasing the ballast coefficients, beam deflection reduces. In the ranges of ballast coefficients 3 to 5 kg/cm^3, bending moments in the rail seat and shear forces are approximately constant and they increase in the ranges of ballast coefficients 5 to 10 kg/cm^3.

1. Introduction

Railway Sleepers, as the foundation of the railway tracks, transfer the train loads to ballast layers. For this reason, analysis and design of railway sleepers as the beam with finite length are important. Analysis and design of railway sleeper can be done based on the beam on the elastic foundation. In this theory, reaction forces of foundation are corresponding to the deflection of each point in the beam. In this model, soil is considered with linear elastic springs [1]. This hypothesis was first developed by Winkler in 1867 and then it was completed by Zimmerman for railway track model [2]. Researchers such as Lee [3], Huang and Thambiratnam [4], Al Nageim et al. [5] and Chen and Yu [6] were developed the Winkler theory for dynamic response of the beam on the elastic foundation. Hetenyi [2] presented relationships for deflection, bending moment and shear forces of beam with infinite, semi-infinite and finite lengths. Also for special cases, Hetenyi [2] presented relationships for all points of the beam by using the superposition principle. In the field of studies related to railway track components including rail, sleeper and ballast, researchers such as Kerr [7] could be pointed out. Zakeri and Abbasi [8] studied the loading pattern variation of concrete sleeper. Also, Zakeri and Abbasi [9] investigated rail bed modulus in the desert area according to field studies. Zakeri and Xia [10] studied the track parameters behavior according to the train–track dynamic interaction model. Moreover, Zakeri et al. [11] studied the effects of train-induced vibrations in desert areas. Milosavljević et al. [12] investigated the effects of vertical track stiffness on the geometry

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deterioration. Moreover, some researchers such as Mohammadzadeh and Mosayebi [13], Ghannadiasl [14], and Uzzal et al. [15] studied responses of the beam on the elastic foundation in railway tracks. In this paper, railway sleeper as a beam with finite length on the elastic foundation is analyzed based on the closed form analytical solution and finite element methods for cases of beam with constant section and without considering the effects of weight. Then, the behavior of B70 concrete sleeper is studied by considering the weight and non-uniform cross-section of sleeper by using the finite element method. In continuation, the effects of ballast coefficients on the behavior of railway sleeper are investigated. Finally, a series of sensitivity analyses are performed on different railway sleeper parameters.

2. Beam on elastic foundation

According to the Euler-Bernoulli beam deformation, the governing differential equation of the beam on the elastic foundation can be considered as follows [1, 2]:

$$EI\left(\frac{d^4y}{dx^4}\right)+ky=q \quad (1)$$

In this equation, parameters of E, I, k, q are Young modulus, moment of inertia of the beam cross section, the coefficient of foundation, and distributed force respectively. Reaction forces are assumed as vertical loads and they are in the opposite direction of beam deformation. So, downward deformation is positive and compression forces create in the foundation. The closed form solution of the mentioned differential equation is as follows:

(2):

$$y=e^{\lambda x}(c_1\cos\lambda x+c_2\sin\lambda x)+e^{\lambda x}(c_3\cos\lambda x+c_4\sin\lambda x)+\frac{q}{k}$$

$$\lambda=\sqrt[4]{\frac{k}{4EI}} \quad (3)$$

Where, "λ" is flexural rigidity of beam on the elastic foundation. Also, constants of equation can be estimated by using geometric and loading conditions of the beam [2]. In addition, slope (θ), bending moment (M), and shear force (Q) of the beam can be calculated from the following relationships:

$$\frac{dy}{dx}=\tan\theta \quad (4)$$

$$-EI\left(\frac{d^2y}{dx^2}\right)=M \quad (5)$$

$$-EI\left(\frac{d^3y}{dx^3}\right)=Q \quad (6)$$

3. Beam with finite length

Figure 1 indicates sleeper with free ends caused by two loads in rail seats. This is what happens for railway sleeper in practical problems. Deflection, slope, bending moment and shear force in different parts of sleeper by using the superposition primary caused by concentrated loads in rail seats are presented in Tables 1 and 2 [2].

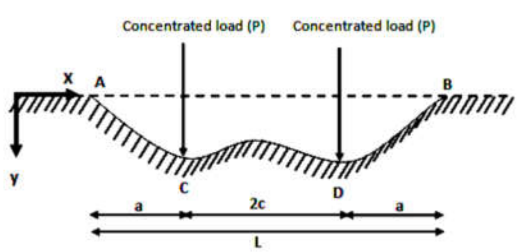

Figure 1. Sleeper as a beam with finite length caused by two loads

In Figure 1, the railway sleeper is divided into three parts including (A-C), (C-D) and (D-B) parts. Tables 1 and 2 present the relationships of different parameters in (A-C) part of sleeper for (x≤a) and (C-D) part of sleeper for (a<x<L-a), respectively [2].

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Table 1. Relationships of different parameters in (A-C) part of sleeper [2]

Parameters	Relationships
Deflection	$y_{A-C}=\frac{P\lambda}{k}\frac{1}{\sinh\lambda l+\sin\lambda l}\begin{bmatrix} 2\cosh\lambda x\cos\lambda x\left[\cosh\lambda a\cos\lambda(l-a)+\cosh\lambda(l-a)\cos\lambda a\right] \\ +(\cosh\lambda x\sin\lambda x+\sinh\lambda x\cos\lambda x) \\ \begin{bmatrix}\cosh\lambda a\sin\lambda(l-a)-\sinh\lambda a\cos\lambda(l-a) \\ +\cosh\lambda(l-a)\sin\lambda a-\sinh\lambda(l-a)\cos\lambda a\end{bmatrix}\end{bmatrix}$
Slope	$\theta_{A-C}=\frac{2P\lambda^2}{k}\frac{1}{\sinh\lambda l+\sin\lambda l}\begin{bmatrix}(\sinh\lambda x\cos\lambda x-\cosh\lambda x\sin\lambda x).\left[\cosh\lambda a\cos\lambda(l-a)+\cosh\lambda(l-a)\cos\lambda a\right] \\ +\cosh\lambda x\cos\lambda x\begin{bmatrix}\cosh\lambda a\sin\lambda(l-a)-\sinh\lambda a\cos\lambda(l-a) \\ +\cosh\lambda(l-a)\sin\lambda a-\sinh\lambda(l-a)\cos\lambda a\end{bmatrix}\end{bmatrix}$
Bending moment	$M_{A-C}=\frac{P}{2\lambda}\frac{1}{\sinh\lambda l+\sin\lambda l}\begin{bmatrix} 2\sinh\lambda x\sin\lambda x\left[\cosh\lambda a\cos\lambda(l-a)+\cosh\lambda(l-a)\cos\lambda a\right] \\ +(\cosh\lambda x\sin\lambda x-\sinh\lambda x\cos\lambda x) \\ \begin{bmatrix}\cosh\lambda a\sin\lambda(l-a)-\sinh\lambda a\cos\lambda(l-a) \\ +\cosh\lambda(l-a)\sin\lambda a-\sinh\lambda(l-a)\cos\lambda a\end{bmatrix}\end{bmatrix}$
Shear force	$Q_{A-C}=p\frac{1}{\sinh\lambda l+\sin\lambda l}\begin{bmatrix}(\sinh\lambda x\cos\lambda x+\cosh\lambda x\sin\lambda x).\left[\cosh\lambda a\cos\lambda(l-a)+\cosh\lambda(l-a)\cos\lambda a\right] \\ +\sinh\lambda x\sin\lambda x\begin{bmatrix}\cosh\lambda a\sin\lambda(l-a)-\sinh\lambda a\cos\lambda(l-a) \\ +\cosh\lambda(l-a)\sin\lambda a-\sinh\lambda(l-a)\cos\lambda a\end{bmatrix}\end{bmatrix}$

Table 2. Relationships of different parameters in (C-D) part of sleeper [2]

Parameters	Relationships
Deflection	$y_{C-D}=\left[y_{A-C}\right]_{x\succ a}+\frac{P\lambda}{k}\left[\cosh\lambda(x-a)\sin\lambda(x-a)-\sinh\lambda(x-a)\cos\lambda(l-a)\right]$
Slope	$\theta_{C-D}=\left[\theta_{A-C}\right]_{x\succ a}-\frac{2P\lambda^2}{k}\sinh\lambda(x-a)\sin\lambda(x-a)$
Bending moment	$M_{C-D}=\left[M_{A-C}\right]_{x\succ a}-\frac{P}{2\lambda}\left[\cosh\lambda(x-a)\sin\lambda(x-a)+\sinh\lambda(x-a)\cos\lambda(l-a)\right]$
Shear force	$Q_{C-D}=\left[Q_{A-C}\right]_{x\succ a}-p\cosh\lambda(x-a)\cos\lambda(x-a)$

In these Tables, y, θ, M, and Q are deflection, slope, bending moment, and shear force of sleeper, respectively.

4. Analysis of railway sleeper

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For analyzing the sleeper behavior as a beam on the elastic foundation, B70 concrete sleeper is considered. Geometrical and mechanical properties of sleeper are considered as follows:

$$l = 260cm \quad a = 55cm \quad b = 30cm \quad c = 75cm \quad h = \ldots \quad \ldots 1E5 \tag{7}$$

Where, L, b and h are dimensions of sleeper. Also, f_c and E_c are mechanical properties of sleeper. In order to validate the results, the obtained results based on the closed form analytical solution and also finite element methods are compared for sleeper with constant section and without effects of weight. Figures 2, and 3 show shear force and bending moment of sleeper, respectively, that are calculated based on the closed form solution and finite element methods. In these Figures, the horizontal axis is distance from the origin of coordinates of sleeper ("A" point in Figure 1).

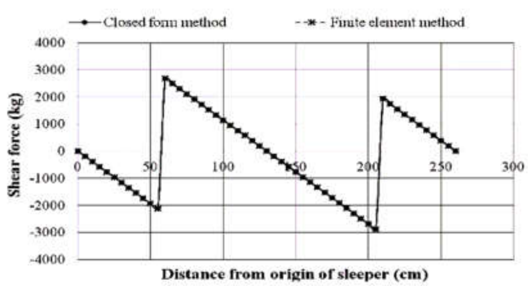

Figure 2. Shear force of sleeper

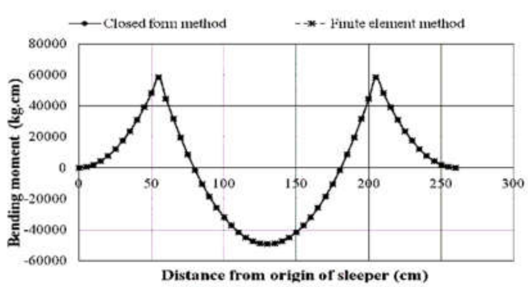

Figure 3. Bending moment of sleeper

As shown from Figures 2 and 3, the obtained results by modeling with the finite element method are same with the results of closed form solution method for beam with constant section and without considering the effects of weight. In continuation, for investigating the effects of weight and non-uniform cross-section of railway sleeper, model of B70 concrete sleeper is considered according to Figure 4 [16].

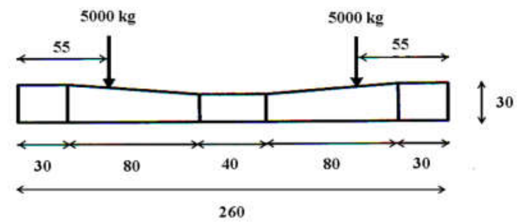

Figure 4. B70 concrete sleeper (Geometric dimensions in centimeters)

In continuation, by using the finite element method, the results are presented. In this regard, the results of the B70 concrete sleeper behavior by considering the effects of weight and non-uniform cross-section are presented in the following Figures.

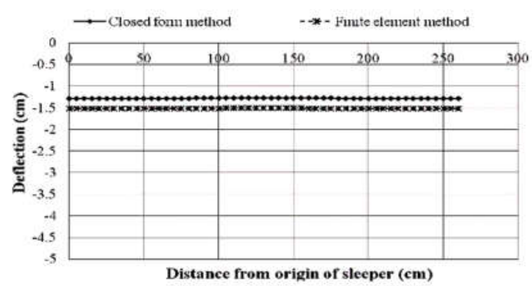

Figure 5. Deflection of sleeper

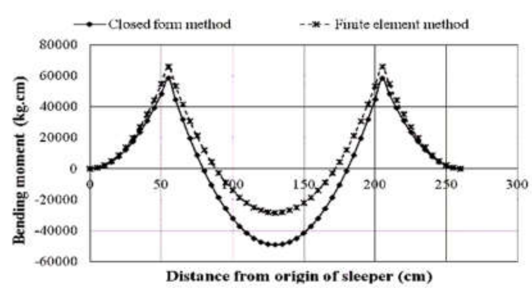

Figure 6. Bending moment of sleeper

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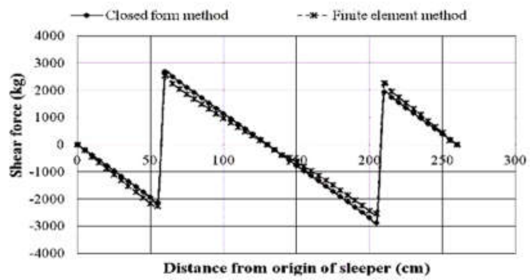

Figure 7. Shear force of sleeper

Figures 5, 6, and 7 present the deflection, bending moment, and shear force of the sleeper respectively that are calculated based on the closed form analytical solution and finite element methods. According to the finite element method and considering two parameters including the weight and non-uniform cross-section of sleeper, deflection increases about 18.42 percent. Also, the bending moment in the middle of sleeper decreases about 42 percent and bending moment in the rail seat increases about 12.77 percent than the closed form analytical solution method. These results indicate that these two parameters have relatively significant effects on the behavior of railway sleeper.

5. Sensitivity analysis on the railway sleeper behavior for different track conditions

Railway tracks of Iran are 9460 km that approximately 425 km of these tracks are in desert areas. In these areas, ballast layers are contaminated by windy sands and consequently, railway ballasted tracks lose their elasticity properties and therefore track stiffness increases. Some destructive effects of windy sands on the railway tracks in desert areas are early wear and deterioration of track components, damage and destruction of concrete sleepers, increase of railway track rigidity, and increase of repair and maintenance operations and costs. Figure 8 shows the destructive effects of windy sands on the ballasted railway tracks.

(a)

(b)

(c)

(d)

Figure 8. Samples of destructive effects of windy sands in desert areas

As can be observed from Figure 8, windy sands cause the damage to railway components. For investigating the behavior of B70 sleeper for different track conditions, the effects of five different types of ballast layer coefficients are studied for analyzing the sleeper behavior. Figures 9, 10, 11, and 12 indicate the deflection of sleeper in the rail seat, bending moment in the middle of sleeper, bending moment in the rail seat, and shear force of sleeper in distance of 20 cm from rail seat, respectively.

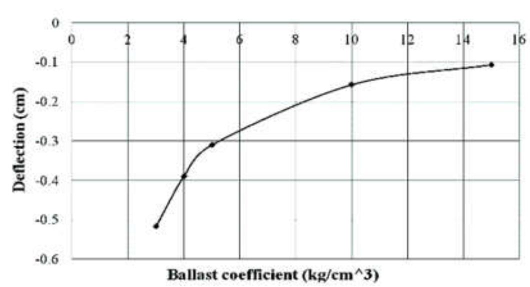

Figure 9. Deflection of sleeper in the rail seat

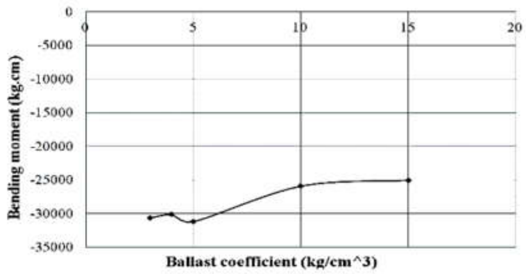

Figure 10. Bending moment in the middle of sleeper

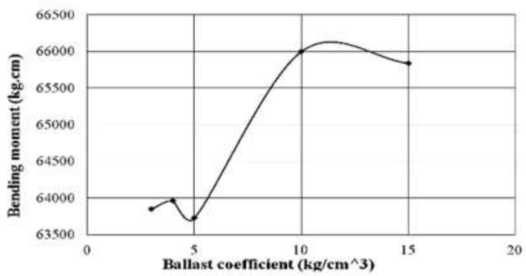

Figure 11. Bending moment in the rail seat

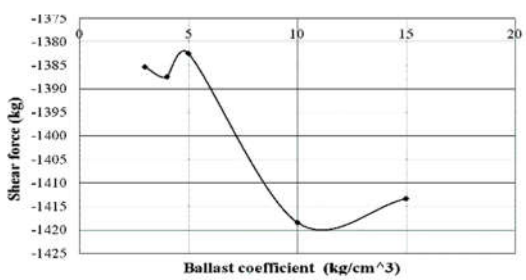

Figure 12. Shear force of sleeper in distance of 20 cm from rail seat

As can be observed from the presented Figures, by considering the effects of windy sands and consequently the increase of ballast layer coefficients, deflection of sleeper reduces. Bending moment of sleeper in the rail seat approximately is constant in the ranges of ballast coefficients 3 to 5 kg/cm^3 and it increases in the ranges of ballast coefficients 5 to 10 kg/cm^3 and again it is approximately constant after ballast coefficient 10 kg/cm^3. Shear force of sleeper in distance of 20 cm from rail seat is approximately constant in the ranges of ballast coefficients 3 to 5 kg/cm^3 and it increases in the ranges of ballast coefficients 5 to 10 kg/cm^3 and then it is approximately constant after ballast coefficient 10 kg/cm^3.

6. Conclusion

According to the railway sleepers play a significant role in the railway tracks, therefore their review, study, analysis, and design are important. In this research, the behavior of railway sleeper as a beam with finite length on the elastic foundation was studied by using analytical and numerical methods. Also, B70 concrete sleeper was analyzed by using the finite element method. Two cases for analyzing the sleeper were considered. The first case was beam without the effect of weight and with a constant cross-section and the second case was beam by considering the effects of weight and with a non-uniform cross-section. Then in continuation, the responses of railway sleeper parameters were calculated according to different ballast coefficients. Important results can be summarized as follow:

- The obtained results by modeling based on the finite element method were the same as results of the closed form solution method for the beam with constant section and without considering the effects of weight.
- By modeling the railway sleeper based on the finite element method and considering the effects of weight and non-uniform cross-section of sleeper, bending moment in the middle of sleeper decreased about 42 percent and bending moment in the rail seat increased about 12.77 percent than the closed form solution method. Therefore, the weight and non-uniform cross-section of sleeper have relatively significant effects on the railway sleeper behavior.
- By increasing the ballast coefficients, bending moment of sleeper in the rail seat approximately was constant in the ranges of ballast coefficients 3 to 5 kg/cm^3 and it increased in the ranges of ballast coefficients 5 to 10 kg/cm^3 and it was approximately constant after ballast coefficient 10 kg/cm^3. Moreover, the shear force of sleeper in distance of 20 cm from rail seat was approximately constant in the ranges of ballast coefficients 3 to 5 kg/cm^3 and it increased in the ranges of ballast coefficients 5 to 10

kg/cm^3 and it was approximately constant after ballast coefficient 10 kg/cm^3.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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